

Arithmetic

Made familiar and easy to
Young Gentlemen and Ladies.

Being the

SECOND VOLUME.

OF THE

Circle of the Sciences, &c.

Published by the KING's Authority.

The SECOND EDITION.

L O N D O N:

Printed for J. NEWBERY, at the *Bible*
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Artistic

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Proposed by the King's University

The Second Edition

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1825



By the KING's Royal Licence.

To His Highness

Prince EDWARD AUGUSTUS,

T H I S

ART of ARITHMETIC

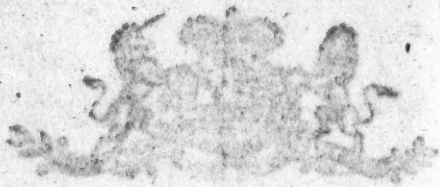
Is humbly Inscrib'd

B Y

His Highness's

most obedient Servant,

JOHN NEWBERRY.



By the King's Royal Licence.



Prince Edward's MUSEUM

THIS

ART OF ARITHMETIC

Is humbly inscrib'd

BY

His Highness's

most obedient servant,

JOHN NEWBERRY.

PREFACE.

I Foresee but one Objection that may be made against the following Compendium of the most useful Rules in Arithmetic, viz. that no Notice is therein taken of the Doctrine of Vulgar Fractions: The Reason of which Omission is, that I thought them too difficult for the tender Capacities of those, for whose Service this little Work is principally intended; and besides, most common Business may be carried on with the Knowledge of Whole Numbers only, or such a small Skill in Fractions as may be soon acquired whenever it is found necessary. In general, I hope the Rules and Examples I have given are laid down and ex-

P R E F A C E.

plain'd in such an easy and familiar Manner, as to fall within the Comprehensions of Youth, and render the Work a proper Introduction to the Study of the more abstruse Parts of Arithmetic. But instead of spending Time in setting forth the Merit of the Performance, (the usual Business of Prefaces) I submit it entirely to the Reader's Judgment, and shall employ a few Pages in giving him a short Account of the Rise and Progress of the Art of Numbers,

We have very little Intelligence indeed about the Origin or Invention of Arithmetic, History neither fixing its Author nor the Time. In all Probability, however, it must have taken its Rise from the Introduction of Commerce, and consequently be of Tyrian Original,

From Asia it passed into Egypt, (by means of Abraham according to Josephus) where it was greatly cultivated and improved, insomuch that a large
Part

P R E F A C E.

Part of their Philosophy and Theology seems to have turn'd altogether upon Numbers: And Pythagoras, who learnt this Science from the Egyptians, attempted to explain all Things in Nature by Numbers, affirming that the Knowledge of Numbers was the Knowledge of the Deity.

From Egypt Arithmetic was transmitted to the Greeks, who handed it forwards, with great Improvements which it had received from their Astronomers, to the Romans; and from them it spread over Europe with their Conquests.

The Arabians, by the happy Invention of the Cypher, made the Way to the Science of Numbers much more easy than it was before; and to them we are indebted for the Art of Algebra, which, after having been long neglected and little understood, has of late been revived and very much improved in Europe.
In

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In truth, the Arithmetic of the Ancients fell far short of that of the Moderns, who in the last and the present Century have applied themselves to the Study of this excellent Science with so much Success, that they seem to have carried it to its utmost Perfection.

As to the Method of Computation in Use amongst us by a Series of ten Characters, so that the Progression is from Ten to Ten, it was utterly unknown to the Greeks and Romans, and is said to have been first introduced into Europe about the End of the tenth Century, by Gerbert, afterwards Pope under the Name of Sylvester II, who borrow'd it from the Moors of Spain. No doubt it took its Origin from the Way of reckoning on the Fingers, which was used before Arithmetic was brought into an Art: And hence perhaps the single Figures, 1, 2, 3, &c. have obtained

P R E F A C E.

and the Name of Digits, from the Latin Word Digitus, a Finger.

The Eastern Missionaries assure us, that to this Day the Indians are very expert at computing on their Fingers, without the Use of Pen and Ink. The Chinese do not much regard Rules in their Calculations, instead of which (Father le Comte tells us) they use an Instrument made of a Plate a Foot and a half long, across which are fitted ten or twelve Iron Wires, whereon are strung little round Balls: By drawing these together, and dispersing them again one after another, they calculate with great Ease and Expedition; and have also their Manner of proving the Truth of the Operation. Add to this, that the Natives of Peru, who make their Computations by the different Arrangement of Grains of Maize, are said to equal any European, with all his Rules, both in Sureness and Dispatch.

I men-

P R E F A C E.

*I mention these Things only as Mat-
ters of Curiosity, and not with any De-
sign to depretiate the noble Science of A-
rithmetic, the Study of which I heartily
recommend to the Youth of both Sexes
as an Attainment that will be of ex-
cellent Service to them almost in every
Station of Life.*

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ARITH

ARITHMETIC.

Its Definition, Usefulness, and principal Rules.

Q. **WHAT** is ARITHMETIC?

A. The *Art of Numbering*, whereby we reckon or calculate truly, and with Ease and Expedition.

Q. Of what Use is it?

A. It is necessary for the carrying on of Trade and Business, for the Management of an Estate, and many other Affairs of Life. Without the Knowledge of *Arithmetic* we are liable to be imposed upon in our Deal-

B

ings

ings with Mankind, or defrauded by careless Servants and unjust Stewards: But by a competent Skill in Numbers these Inconveniencies may be avoided, and we may probably increase our Store, and become rich and honourable. Even the Ladies themselves, who have generally the Care of the domestic Expences of a Family, ought therefore to have a proper Share of this useful Accomplishment.

2. Which are the chief Rules in Arithmetic?

A. *Numeration, Addition, Subtraction, Multiplication, and Division.*

CHAP. I.

Of NUMERATION, or NOTATION.

2. WHAT is NUMERATION?

A. By *Numeration*, or *Notation*, (as it is call'd by some late Arithmeticians) we are taught how to read or write down any Number proposed.

2. How

Q. How is that to be done?

A. By the ten following Figures or Characters, *viz.*

One, Two, Three, Four, Five,

¹ ² ³ ⁴ ⁵
Six, Seven, Eight, Nine, Cypher.

⁶ ⁷ ⁸ ⁹ ⁰

Q. But how can these few Figures expresse all Numbers whatsoever?

A. By having different Values or Significations set upon them according to the Place they stand in; for though each of them when alone has a fix'd and certain Value, as 1 *one*, 2 *two*, &c. yet the same Figure, when placed along with others, may expresse very different Numbers; as 1 may signify *one* only, or *ten*, or a *hundred*, a *thousand*, *ten thousand*, &c.

Q. How then shall I know the Value of any Figure?

A. By observing the Place it stands in, and remembring, that if it be a-

B 2

lone,

lone, or in the first Place on the Right Hand, it signifies so many *Units* or *Ones* as the Figure simply expresse; as, 5 denotes five *Units*: But place another Figure after it, and it signifies five *Tens*, or *Fifty*; as 51 is *Fifty-one*. So if another Figure were added after it, the 5 would signify five *Hundreds*, that is, just ten Times as many as it did before; for each Remove from the Right Hand towards the Left increases the Value of a Figure in a ten-fold Proportion, from Ones to Tens, from Tens to Hundreds, from Hundreds to Thousands, from Thousands to Tens of Thousands, &c.

Q. What is the Use of the round Character you call a *Cypher*?

A. Though of itself it signifies *Nothing*, yet being placed after other Figures it gives Value to them in the above-mention'd Proportion: Thus 9, when alone, stands for *Nine* only; but
being

being removed one Step towards the Left Hand by adding a Cypher after it, it signifies ten times as many, that is, 90, *Ninety*. But what has been said will appear more plainly by a due Attention to the following Table.

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
							1	1
						1	2	2
					1	2	3	3
			1	1	2	3	4	4
		1	2	2	3	4	5	5
	1	2	3	3	4	5	6	6
1	2	3	4	4	5	6	7	7
				5	6	7	8	8
					6	7	8	9

B 3

From

From this Table you may observe, that 1 standing alone signifies a simple Unit, or *One* only; but in the next Row of Numbers, being placed on the Left of the Figure 2, it stands for one *Ten* or ten Units, and both of them put together make up *Twelve*. In the next Row you find the Figure 1 signifies *one Hundred*, being remov'd into the third Place from the Right Hand, with the Figure 2 in the Place of Tens, and 3 in that of Units: The Sum of these three Figures therefore, as they stand in a Row, is 1 Hundred, 2 Tens or Twenty, and 3 Units or Ones; that is, *One hundred and twenty-three*. Again: In the next Row of Figures the 1 is in the fourth Place from the Right Hand, which makes it stand for *One Thousand*, having 2 after it in the Place of Hundreds, 3 in the Place of Tens, and 4 in that of Units; which express'd together, as they stand

in

in a Row, 1234, amount to *One thousand two hundred and thirty-four*. Thus exercising yourself at first in the smaller Numbers, you may proceed to the greater till you are perfect in the whole Table, observing how the Value of Figures increases as they are removed from the Right Hand towards the Left, and remembering the Names of the Places: And by this means you will soon be able to express in Words, or to write down in Characters, any Number you will have Occasion for in the common Business of Life.

2. But how shall I know how to write in Figures any Number that is not in the Table, suppose it be *Two hundred and twenty five*?

A. As you begin with Hundreds, which you see by the Table must stand in the *third* Place towards the Left Hand, it is plain that three Figures will be necessary to express the Num-

ber proposed. Therefore first set down 2 for the Hundreds; after that 2 for the Tens, because there are two Tens in Twenty; then 5 in the Place of Units, and the Figures will express the Number you desire, 225.

Q. If Cyphers be mix'd amongst Figures, how must they be reckon'd?

A. Remember that of themselves they signify *nothing*, but increase the Value of other Figures that stand before them: Suppose therefore you meet with this Number 400, as there is a Cypher in the Place of Units, and another in the Place of Tens, these are of no Signification; but the 4 denotes *four hundred*, being advanced to the Place of Hundreds by the two Cyphers. So if you would express *two hundred and five* in Figures, you must set down 2 for the Hundreds, and having no Tens make a Cypher in the Place of Tens, and a 5 in that of Units,

nits, and you have the Number proposed, 205.

Q. How is the lowest Row of Figures in the Table (*viz.* 123456789) to be express'd in Words at length?

A. *One hundred twenty-three Millions, four hundred fifty-six Thousand, seven Hundred eighty-nine.* N. B. In these long Numbers it is usual to make a Comma or Stop between every three Figures, from the Right Hand to the Left, whereby the Millions, Thousands, &c. are more easily distinguish'd; as,

231,564,970
24,301,025
5,210,344

Which the young Scholar may endeavour to express in Words for his Improvement, after he is well acquainted with smaller Numbers.

CHAP. II.
Of ADDITION.

2. **W**HAT is ADDITION?

A. It is the *putting together of several Numbers, so as to find their total Amount, that is, one Number equal to them all.*

2. What is first to be observ'd in this Operation?

A. In setting down the Numbers to be added, Care must be taken to place every Figure in its proper Column, that is, Units under Units, Tens under Tens, Hundreds under Hundreds, &c. as in the following Examples:

<i>Books.</i>	<i>Nuts.</i>	<i>Pins.</i>
1	24	172
7	80	345
8	61	201
3	43	129

In all 19

In all 208

In all 847

2. In what Manner do you add these Numbers together?

A. The first Example, or that on the Left Hand, is a Row of single Figures, which I sum up in this Manner: Beginning at the Bottom, and going upwards, I say, 3 and 8 is 11, and 7 is 18, and 1 is 19; and having no more Figures to add, I set down 19 at the Bottom as the total Amount.

—In the second Example I likewise begin at the Bottom of the Rank of Units (which is always to be done) and say, 3 and 1 is 4, and a Cypher is *Nothing*, and 4 is 8, which being less than

than 10 I set down under the Units, and proceed to the Left-hand Column, saying, 4 and 6 is 10, and 8 is 18, and 2 is 20; which I put down underneath on the Left of the 8, and find the Amount of the several Parts or Figures to be 208.—In like manner I begin with the Units of the third Example, saying, 9 and 1 is 10, and 5 is 15, and 2 is 17; which being 7 more than 10, I set down 7 under the Row of Units, and carry 1 for the 10 to the next Column on the Left, saying, 1 and 2 is 3, the Cypher is Nothing, and 4 is 7, and 7 is 14. Here again, the Number 14 being 4 more than 10, I put down 4 under the Row of Tens, and carry 1 to the Column of Hundreds, saying, 1 and 1 is 2, and 2 is 4, and 3 is 7, and 1 is 8, which 8 I place underneath, and find the Total to be 847.

Q. Is *One* for *Ten* to be always carried from any Row of Figures to the next on the Left Hand, as in the last Example?

A. In the Addition of Numbers of one Denomination, that is, of one Name or Kind, whether they be all *Gallons, Pounds, Miles, Acres*, or whatever, (which is call'd *Simple Addition*) you must always carry 1 for every 10 that you find in the Row of Units to the next Row of Tens, and the like from the Tens to the Hundreds, from the Hundreds to the Thousands, &c. till you have gone through all the Rows, be they ever so many. So if the Amount of the Column of Units be just 10 and no more, set down a Cypher underneath the Units, and carry 1 to the Tens; but if they amount to 20, a Cypher must be put down, and 2 carried, because there are two Tens in Twenty. In like manner,

manner, if the Amount of a Right-hand Column be 45, the 5 must be placed underneath in the Total, and 4 carried to the Column on the Left, because there are four Tens in Forty. When you come to the last Column, the Amount of it must be set down, be it what it will, because the Operation is compleated, and the Tens can be carried no farther. Take an Example or two more for Practice.

<i>Boys.</i>	<i>Horses.</i>
2907	4572
315	8601
8549	209
2660	74
Total 14431	13456

Let us now proceed to *Compound Addition*, or that of Numbers of different Denominations, as we find in *Money, Weight, Measure, &c.* And first of *Money.* 2. What

2. What is to be observ'd in casting up Sums of Money?

A. Remember that in *England* we keep our Accounts in *Pounds, Shillings, Pence, and Farthings*; and therefore take particular Notice, that

4 Farthings	} make	{	1 Penny.
12 Pence			1 Shilling.
20 Shillings			1 Pound.

Besides, for your greater Ease in this Sort of Addition, it will be proper to get by heart the following

PENCE-TABLE.

20d.	is	1s.	8d.
30	—	2	6
40	—	3	4
50	—	4	2
60	—	5	0
70	—	5	10
80	—	6	8
90	—	7	6
100	—	8	4
110	—	9	2
120	—	10	0

Q. What is farther to be observ'd?

A. In setting down the Numbers to be added together, be sure to place Pounds under Pounds, Shillings under Shillings, Pence under Pence, and Farthings under Farthings; and you may put an *l.* over the Column of Pounds, an *s.* over the Shillings, a *d.* over the Pence, and a *q.* or *qr.* over the Farthings.

Q. What is the Meaning of those Letters?

A. They are the first Letters of the Latin Words *Libra* a Pound, *Solidus* a Shilling, *Denarius* a Penny, and *Quadrans* a Farthing.

Q. Let me see an Example according to these Directions.

A. I owe to one Person *Three Pounds six Shillings and Sixpence Farthing*, to another *One Pound four Shillings and Tenpence Half-penny*, and to a third *Two Pounds eleven Shillings and three*

three Farthings. In order to know what these several Debts amount to, I set them down in the following Manner, and find the Total to be Seven Pounds two Shillings and Five-pence Halfpenny.

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
3	6	6	1
1	4	10	2
2	11	0	3
7	2	5	2
Total.			

Q. But how do you proceed in the Operation?

A. I begin with the Farthings, and say, 3 and 2 is 5 and 1 is six; then considering that 4 Farthings make 1 Penny, I set down only 2 Farthings under that Column, and carry 1 for the 4 to the Column of Pence. Then (passing over the Cypher) I say, 1 that I carry and 10 is 11, and 6 is 17; which

which being 5 Pence more than a Shilling, I set down 5 under the Pence and carry 1 for 12 to the Column of Shillings; saying, 11 that I carry and 11 is 22, and 4 is 16, and 6 is 22. Here again I consider that 22 Shillings is 2 more than a Pound; therefore I set down 2 under the Shillings, and for the 20 I carry 1 to the Column of Pounds; saying, 1 I carry and 2 is 3, and 1 is 4, and 3 is 7, which I set down under the Pounds, and so the Work is compleated.

Q. Is the same Method to be observ'd in all Sums of this Nature?

A. Yes; always remember for every four Farthings to carry 1 Penny, for every 12 Pence to carry 1 Shilling, and for every 20 Shillings to carry 1 Pound; setting down, as you go along, the overplus Farthings, Pence, and Shillings (if such there be) under their proper Columns.

2. Are

2. Are Farthings always express'd as in the Example, with a *q.* over them?

A. No; they are commonly set closer to the Column of Pence, and are express'd as follows, *viz.* $\frac{1}{4}$ is one Farthing, $\frac{1}{2}$ is two Farthings or an Halfpenny, $\frac{3}{4}$ is three Farthings. These are call'd *Fractions*, and are to be cast up in the same Manner as before; as you will find in these Examples:

	<i>s.</i>	<i>d.</i>
Laid out for Apples	1 —	6 $\frac{1}{2}$
Gingerbread	2 —	4 $\frac{3}{4}$
Marbles —	0 —	9
Oranges —	1 —	5 $\frac{1}{4}$
Total	6 —	1 $\frac{1}{2}$

Here I cast up the Farthings, as in the former Example; and finding them to be 6, or 2 more than a Penny,

C 2

I set

I set down 2 Farthings or a Halfpenny thus $\frac{1}{2}$, and carry 1 to the Pence, which being added together amount to 25. Then considering that in 25 Pence there are twice 12, (that is, two Shillings) and 1 Penny over, I put down the odd Penny under the Column of Pence, and carry 2 to the Shillings; which amounting to 6, I place 6 under that Column, and find the Total to be as above express'd.—Take one Example more for your own Practice.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Laid out	22	— 16	— $8\frac{1}{4}$
and	33	— 10	— 4
and	29	— 3	— $7\frac{3}{4}$
Total	85	— 10	— 8

N. B. When the Column of Pounds has two or more Figures in a Row,

Row, add them up as Numbers of one Denomination ; that is, for every 10 in the Place of Units carry 1 to that of Tens, and the like from the Tens to the Hundreds, &c.

Q. How shall I learn the Addition of *Weight*?

A. Two Sorts of Weight are used in *England*, the one call'd *Avoirdupois*, the other *Troy*; both which ought to be understood. We shall begin with the *Avoirdupois* Weight, as being the most common; the several Divisions whereof are as follows, and must be well remember'd:

AVOIRDUPOIS *Weight*.

16 Drams *make* 1 Ounce.
 16 Ounces ——— 1 Pound.
 28 Pounds ——— 1 Quarter of a Hund.
 4 Quarters — 1 Hundred.
 20 Hundreds — 1 Tun.

In *setting down* Sums of this Kind, always begin with the Numbers of the highest Denomination, proceeding from the Left Hand to the Right; but in *casting them up* begin with those of the lowest Denomination, proceeding from the Right Hand to the Left. As in this Sort of Weight, first put down *Tuns*, then *Hundreds*, then *Quarters*, &c. but in adding them together do quite the Reverse, that is, begin with the smallest Divisions, and end with the greatest.

Q. What Marks are set over the Columns in this Sort of Weight?

A. *T.* is used for *Tuns*, *C.* for *Hundreds*, *qrs.* for *Quarters*, *lb.* for *Pounds*, *oz.* for *Ounces*, and *dr.* for *Drams*.

Q. In adding up these Sums, what is to be carried from one Column to another?

A. For every 16 *Drams* carry 1 *Ounce*, for 16 *Ounces* carry 1 *Pound*,
for

for 28 Pounds carry 1 Quarter, for 4 Quarters carry a Hundred, and for 20 Hundreds carry 1 Tun.

Q. Can't you give me an Example.

A. Yes, First let us set down one in *Great Weight*, that is, where the Columns are *Tuns, Hundreds, Quarters,* and *Pounds.*

T.	C.	qrs.	lb.
2	5	1	16
3	4	3	10
1	7	2	03
2	9	1	22
9	7	0	23

Here I begin to cast up the Column of Pounds, which I find amount to 51, and remembring that for every 28 I must carry 1 to the Column of Quarters, I consider how often 28 is contain'd in 51; which being but once,

C 4

and

and 23 over, I set down 23 under the Pounds, and carry 1 to the Quarters. Upon summing up these I find them to be just 8, and as I am to carry 1 for every 4, I put down 0, and carry 2 to the Hundreds. These being added together make 27; and as I am to carry 1 for every 20, I set down 7 under the Hundreds, and carry 1 to the Tuns; which amounting to 9, I set it down underneath, and the Total appears as above.

✿ It may not be amiss to observe, that though some Goods are sold by the Tun, they are not weigh'd by it, but by Hundreds, Quarters, and Pounds, and the Tuns are computed afterwards.

The following is an Example of the *Small Weight*, consisting of *Pounds, Ounces, and Drams*.

[25]

<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
21	— 11	— 10
15	— 08	— 12
22	— 12	— 14
37	— 09	— 03
97	— 10	— 07

In this Example, remembering that I am to carry 1 for every 16 Drams, and the same for every 16 Ounces, I begin with casting up the Drams, and find they amount to 39, which Number contains twice 16, and 7 over: I therefore set down 7 at the Bottom, and carry 2 to the Ounces, which I add together, and find them amount to 42. Now the Number 42 containing twice 16 and 10 over, I put down 10, and carry 2 to the Pounds, which making 97 in all, I set down 97, and the Work is finish'd.

Q. Can't you give me an Example of the *Great* and *Small* Weights together?

A. Yes; the following may serve to employ you when you perfectly understand the former:

	(20)	(4)	(28)	(16)	(16)
<i>Tuns.</i>	<i>C.</i>	<i>qrs.</i>	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
753	— 19	— 3	— 27	— 15	— 15.
347	— 08	— 1	— 17	— 10	— 06.
283	— 11	— 0	— 12	— 03	— 10
549	— 05	— 2	— 13	— 09	— 07.
251	— 13	— 0	— 15	— 03	— 11
2185	— 18	— 1	— 02	— 11	— 01

The Figures at top within Parentheses are placed there to assist the Memory, being the Numbers for which 1 is to be carried to the next Column on the Left, as often as they are contained in the Column over which

which they stand. It is not necessary to place such a Direction over the last Column on the Left, because that is always to be cast up as Numbers of one Denomination, carrying 1 for every 10 Units to the Place of Tens, and in the same Proportion from the Tens to the Hundreds, &c. as was taught in the former Part of *Addition*.—This Method may be observ'd in all Sums of the like Nature, if the Learner want such Assistance.

Q Is there no other Help for the Memory in casting up large Sums?

A. Yes; in casting up a long Column or Row of Figures, you may make a Dot with your Pen as often as you come to the Number for which you are to carry 1 to the next Column, and set down the Overplus at the Bottom. This is done in the Column of Drams in the last Example, where 16 is the Number for which 1 is to be carried

carried to the Column of Ounces. casting it up I say, 11 and 7 is 18 which being 2 more than 16, I make a Dot against the 7, and say, 2 and 12 is 14 and 6 is 20; I therefore make another Dot against the 6, and carry on the 2, which with 15 makes 17 that is, 1 more than 16: And having no farther to go, I set down the overplus 1 at the Bottom, and for the three Dots I carry 3 to the next Column. Thus if you were to cast up a long Row of Pence, you must make a Dot at every 12, and setting down the overplus above the Twelves, carry for each Dot to the Column of Shillings.—And this Method may be used in all Cases of the like Nature wherever it contributes to Ease or Expedition.

Q. What Sorts of Goods are weighed by *Avoirdupois* Weight?

A. All Sorts of Grocery Ware
Butcher

s. Butchers Meat, Bread, Butter, Cheese,
is tallow, Soap, Hemp, Flax, Iron,
malen, Copper, Lead, Pitch, Tar, and
nd other gross and coarse Goods in ge-
maleral.

can 2. What is the Difference between
s 1 *Troy Weight* and *Avoirdupois*?

aving A. This will appear by the follow-
overing Table of

TROY Weight.

mn. 4 Grains make 1 Penny-weight.
a long 16 Penny-weights 1 Ounce.
a D 12 Ounces ——— 1 Pound.

ne C But observe, that the *Troy Ounce* is
arry somewhat larger than the *Avoirdupois*,
F Sh 4 Ounces 12 Penny-weights of the
y l former being equal to 16 Ounces of
atur the latter. — Take notice also, that
or E Grains are denoted by *grs.* and Pen-
igh ny-weights by *pwt.* or *dwt.*; the
Pounds and Ounces as before. See
7 are an Example.

[30]

lb. oz. dwts. grs.

27 — 14 — 19 — 23

39 — 19 — 17 — 19

46 — 09 — 13 — 22

13 — 10 — 08 — 11

158 — 07 — 00 — 03

Q. How do you proceed in casting up *Troy Weight*?

A. Begin with the Grains, and for every 24 carry 1 to the Penny-weights for every 20 Penny-weights, carry 1 to the Ounces, and for every 12 Ounces carry 1 to the Pounds.

Q. What Things are weigh'd by *Troy Weight*?

A. Gold, Silver, Jewels, Amber, Electuaries, &c. and also Liquors are computed by this Weight, a Pint of Wine; Water, &c. being a Pound.

N. B. Twenty-five *lb. Troy* is a *Quar*

of a Hundred.—It is likewise to be noted, that the Apothecaries make use of the *Troy* Pound, Ounce, and Grain, and divide the Ounce into 8 Drams, the Dram into 3 Scruples, and the Scruple into 20 Grains; as in the following Table.

APOTHECARIES *Weight.*

20 Grains *make* 1 Scruple.

3 Scruples — 1 Dram.

8 Drams — 1 Ounce.

12 Ounces — 1 Pound.

Here observe, that though Apothecaries compound their Medicines by this Weight, they buy and sell their Drugs by *Avoir du pois*.

Q. How do you carry in casting up of this Weight?

A. For every 20 Grains carry 1 to 3 Scruples, for every 3 Scruples carry 1 to the Drams, for every 8 Drams carry 1 to the Ounces, and for every 12 Ounces

12 Quinces carry 1 to the Pounds.

2. What are the *Measures* used in *England*?

A. There are various Sorts, as you will learn from the ensuing Tables.

WINE Measure.

2 Pints *make* 1 Quart.

4 Quarts — 1 Gallon.

63 Gallons — 1 Hogshead.

2 Hogsheads - 1 Pipe or Butt.

2 Pipes — 1 Tun.

In adding this Measure, for every 4 Quarts carry 1 to the Gallons, for every 63 Gallons 1 to the Hogsheads, for every 2 Hogsheads 1 to the Pipes, and for every 2 Pipes 1 to the Tuns — *N. B.* The Wine Gallon contains 231 cubic or solid Inches, by which all Liquids are measured, except Beer and Ale.

BEER and ALE Measure.

2 Pints	make	1 Quart.
4 Quarts	—	1 Gallon.
9 Gallons	—	1 Firkin.
2 Firkins	—	1 Kilderkin.
2 Kilderkins	—	1 Barrel.
3 Barrels	—	1 Butt.

The Beer and Ale Gallon are the same, containing 282 solid Inches; but take notice, that 8 Gallons make 1 Firkin of Ale, and consequently a Barrel of Ale is 4 Gallons less than a Barrel of Beer.—In adding this Measure, for every 9 Gallons carry 1 to the Firkins, and for every 4 Firkins carry 1 to the Barrels.—N. B. One Barrel and a half, or 54 Gallons, is a *hogshead* of Beer.

DRY Measure.

8 Pints	make	1 Gallon.
2 Gallons	—	1 Peck.
	D	4 Pecks

4	Pecks	—	1	BusHEL.
4	BusHels	—	1	Coom.
2	Cooms	—	1	Quarter.
5	Quarters	—	1	Wey.
2	Weys	—	1	Last.

N. B. Four Quarters are commonly call'd a Chaldron, but in *London* the Chaldron of Coals is 36 Bushels.—
adding, for every 2 Gallons carry
to the Pecks, for every 4 Pecks carry
1 to the Bushels, for every 8 Bushels
1 to the Quarters, &c.

LONG Measure.

3	Barley-Corns	make	1	Inch.
12	Inches	—	1	Foot.
3	Feet	—	1	Yard.
5	Yards and $\frac{1}{2}$	—	1	Pole or Perch.
40	Poles	—	1	Furlong.
8	Furlongs	—	1	Mile.

In adding this Measure, for every
12 Inches carry 1 to the Feet,
every 3 Feet 1 to the Yards, &c.

N. B. Yards are sometimes set
down in Half-Yards, and Feet in Inches;
and then for every 18 Inches, 1
is carried to the Half-Yards, and for
every 11 Half-Yards 1 to the Poles.

LAND or SQUARE Measure.

40 Square Poles *make* 1 Rood.

4 Roods ——— — 1 Acre.

In this Measure carry 1 for every
40 Poles, and 1 for every 4 Roods.

CLOTH Measure.

4 Nails *make* 1 Quarter.

4 Quarters ——— 1 Yard.

5 Quarters ——— 1 Ell *English*.

3 Quarters ——— 1 Ell *Flemish*.

In adding, for every 4 Nails carry
to the Quarters, for every 4 Quar-
ters carry 1 to the Yards; for every
Quarters carry 1 to the Ells if *Eng-*
ish, and for every 3 if *Flemish*.

D 2

TIME.

TIME.

60 Seconds *make* 1 Minute.

60 Minutes ——— 1 Hour.

24 Hours ——— 1 natural Day.

7 Days ——— 1 Week.

4 Weeks — — — 1 Month.

13 Months, 1 Day, }
6 Hours. } 1 Solar Year.

But the Year is commonly divided in to 12 Calendar Months, some containing 30 Days, others 31, amounting in all to 365. *Note*, however that the true Solar Year is equal to 365 Days, 5 Hours, 49 Minutes, Seconds, and 21 Thirds. — In the Addition of Time, for every 60 Seconds carry 1 to the Minutes, for every 60 Minutes carry 1 to the Hours, &c.

I think it needless to set down Examples of these several Measures, if those of *Money* and *Weights* be well understood, and these Tables carefully attended

tended to, the Scholar will soon know how to *carry* from one Denomination to another, and consequently be able to cast up any Sum whatsoever in *Compound Addition*. I shall therefore only give the *Proof* of this Rule, and proceed to *Subtraction*.

Proof of Addition.

The best Way of proving *Addition*, whether *Simple* or *Compound*, is to begin at the Top of the Sum, and reckon the Figures downwards, in the same Manner that they were added upwards; and if the second Total be the same as the first, the Operation has been perform'd right, otherwise there is some Mistake.

CHAP. III.

Of SUBTRACTION.

WHAT is SUBTRACTION?

A. It is the taking a lesser

D 3

Number

Number from a greater, in order to find the Difference between them.

Q. What is to be observ'd in preparing Numbers for this Operation?

A. Set down the lesser Number under the greater, and let Units stand under Units, Tens under Tens, &c. in the same Manner as if they were prepared for Addition.

Q. What is the next Thing to be done?

A. Having drawn a Line underneath the Numbers, begin at the lower Figure in the first Column to the Right Hand, and subtract it from the upper one, setting down the Remainder under the Line; and so proceed towards the Left, subtracting every lower Figure in each Column from that above it, *i. e.* Units from Units, Tens from Tens, Hundreds from Hundreds, &c. and putting the several Remainders un-

de

er the Line in their correspondent Places.

2. Is there nothing farther to be observ'd in subtracting?

A. When the upper Figure is less than the lower, and consequently the lower cannot be subtracted from it, add 10 to it, which is call'd *borrowing* so many from the next significant Figure on the Left Hand in the upper Line; and for the 10 so borrow'd you must pay 1 to the next Figure or Cypher on the Left Hand in the lower Line: As will be made plainer by Examples.

Suppose I buy 325 Pens, and sell 236, how many shall I have remaining?

	<i>Pens.</i>
Bought	325
Sold —	236
Remain	089

D 4

Here

Here I begin with the lowest Figure in the first Column on the Right Hand, saying, 6 from 5 I cannot take, but (borrowing 10 and putting it to the 5) 6 from 15 and there remains 9, which I set down underneath, and for the 10 that I borrow'd and added to the 5 in the upper Line, I add 1 to the 3 in the lower Line, saying, 1 that I borrow'd and 3 is 4; but not being able to take 4 from 2, I again borrow 10, and say, 4 from 12 and there remains 8, which I set down, and for the 10 that I borrow'd I add 1 to the 2, saying 3 from 3 and there remains 0, which I put down under the last Column, and find the Remainder to be 89, as you see in the Example.

Again: Let it be required to take the Number 3082 from 4205, I set down the Figures as above directed.

From

[41]

From	4205
Take	3082
Rem.	1123

In this Example, beginning with the Column of Units as usual, I say, 2 from 5 and there remains 3, which I place underneath and proceed; 8 from 0 I cannot, but 8 from 10 (which I borrow) and there remains 2: This being set down, I say, 1 that I borrow'd and 0 is 1; which take from 3 and there remains 1; then 3 from 4, and 1 also remains. Thus I find the Difference between the two given Numbers to be 1123.

The Number of Years elapsed since any Year past may be known by subtracting the Date of the past Year from the Date of the present. Suppose then I would know how many Years it is since the Gun-powder Plot, which
happen'd

happen'd in the Year 1605, I set down the Date of the present Year 1745, and the other under it, and then subtracting in the Manner already shewn, I find the Difference to be 140, the Number of Years required.

Present Year	1745
Year of the Plot	1605
Years since	. 140

2. How do you proceed in subtracting Numbers of divers Denominations.

A. Observe this general Rule, That whatever Number you *carried for* in Addition, the same Number you must *borrow* in Subtraction, if it be wanted. — For Instance, in adding Pence together, for every 12 you carried 1 to the Shillings; and so in subtracting Pence from Pence you must borrow 12 from the Shillings, that is, 12 Pence,

Pence, when Occasion requires. And this you must always pay to the next Place on the Left, by adding 1 to the Figure in the lower Line.—Here follows an Example in the Subtraction of Money.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Borrow'd	242	— 16	— 3 $\frac{1}{4}$
Paid	174	— 17	— 9 $\frac{1}{2}$

Remains to pay 67 — 18 — 5 $\frac{3}{4}$

In this Example I begin with the Farthings, saying, 2 from 1 I cannot take, but borrowing a Penny, that is 4 Farthings, and adding it to the 1, I say, 2 from 5 and there remains 3 Farthings, which I set down thus $\frac{3}{4}$, as you see above. Then for the 4 Farthings that I borrow'd I add 1 to the 9 in the Column of Pence, saying, 1 and 9 is 10, which I cannot take from 3: but borrowing 12 Pence,

(i.e.

(i. e. a Shilling) and adding them to the 3, I say, 10 from 15 and there remains 5; which I put down under the Pence, and proceed, saying, 1 that I borrow'd (that is, 1 Shilling) and 17 is 18, 18 from 16 I cannot, but borrowing 20 Shillings, (or 1 Pound) and adding them to the 16, which make 36, I say, 18 from 36 and there remains 18: This I set down under the Shillings, and for the 20 Shillings borrow'd I add 1 to the Pounds, saying, 1 and 4 is 5, 5 from 2 I cannot, but borrowing 10 from the next Place, and adding it to the 2, I say, 5 from 12 and there remains 7, which I put down under the Units, and for the 10 I borrow'd I add 1 to the 7 in the lower Line, saying, 8 from 4 I cannot, but 8 from 14 (borrowing 10 more) and there remains 6; then 1 for the 10 I borrow'd and 1 is 2, 2 from 2 and there

there remains nothing. Thus the Work is compleated, and the Money still to be paid appears to be *Sixty-seven Pounds, eighteen Shillings, and Five-pence three Farthings.*—*N. B.* In subtracting the Pounds we borrow by Tens, as in Numbers of one Denomination; and such the last Column on the Left Hand is always to be reckon'd, let its Name be what it will.

Another Example.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
From	1296	10	$10\frac{1}{2}$
Take	873	5	$7\frac{1}{4}$
Rem.	423	5	$3\frac{1}{4}$

Q. What is to be observ'd in the Subtraction of *Weight*?

A. In the Subtraction of *Avoirdupois* Weight, in case of Want in the Drams or Ounces borrow 16, in the Pounds

Pounds 28, and in the Quarters 4. In Troy Weight, when Necessity requires, borrow 24 in the Grains, 20 in the Penny-weights, and 12 in the Ounces; always remembering to pay 1 for what you borrow to the next Place on the Left Hand in the lower Line. And you may set these Numbers over the Columns to assist your Memory if you think proper, as you see in the following Examples:

AVOIRDUPOIS.

	(4)	(28)	(16)	
	C.	grs.	lb.	oz.
From	16	—3—	13—	9
Take	8	—1—	21—	12
Rem.	8	—1—	19—	13

Here, beginning with the Ounces, I say, 12 from 9 I cannot, but 12 from 16 and there remains 4, which added

added to 9 makes 13. I therefore set down 13 under the Ounces, and for the 16 that I borrow'd I add 1 to the Pounds, saying, 22 from 13 I cannot, but 22 from 28 and there remains 6, which with 13 makes 19. I then set down 19, and for the 28 that I borrow'd I add 1 to the Pounds, saying 1 and 1 is 2; 2 from 3 and there remains 1; which being put down under the Quarters I proceed, saying, 8 from 16 and there remains 8: And this being set down under the Hundreds the Operation is finish'd.

TROY.

(12) (20) (24)

lb. oz. dwt. gr.

From 62—4—10—11

Take 41—9—06—16

 20—7—03—19

In this Example I say, 16 from 11 I cannot take, but 16 from 24 and there remains 8, which added to 11 makes 19. Then setting down 19 under the Grains, for 24 I borrow'd I add one to the Penny-weights in the lower Line, saying, 1 and 6 is 7, which take from 10 and there remains 3. I then put down 3 and proceed saying, 9 from 4 I cannot, but 9 from 12 and there remains 3 and 4 is 7; which being set down I carry 1 to the Pounds, saying, 1 and 1 is 2, 2 from 2 and there remains 0, 4 from 6 and there remains 2. Thus the Work is compleated, and stands as you see above.

Q. How do you proceed in the Subtraction of *Measure*?

A. In the same Manner as you have seen in the Examples of *Weights* and *Money*; for in all Kinds of *Compound* Subtraction the chief Business is

to remember, that the Number borrow'd in any Column is always so many as will make 1 in the next; and how many of a lower Denomination make one of a higher may be learnt from the foregoing Tables.

2. How do you prove that your Work is perform'd right?

A. The Proof of Subtraction is very easy; namely, by adding the Remainder to the Sum subtracted, and, if there be no Mistake, the Total will be the same with the uppermost Line, from whence the Subtraction was made; as appears in the following Instance.

	<i>l.</i>	<i>s.</i>	<i>d.</i>	
From	42	16	09	
Take	18	16	11	
Rem.	23	19	10	
Proof	42	16	09	

} Add

E

I shall

I shall conclude this Rule with one Example, from whence the Learner may discern how all Questions of the same Nature may be answer'd. Suppose I have already saved up 39*l.* 18*s.* and 10*d.* how much Money do I want to make up a hundred Pounds?

	<i>l.</i>	<i>s.</i>	<i>d.</i>
From	100	00	00
Take	39	18	10
Answer	60	01	02
	100	00	00

CHAP. IV.

Of MULTIPLICATION.

QU**W**HAT is MULTIPLICATION?

A, It is a Rule whereby from two Numbers given we find out a third which contains either of the two Numbers

ers as often as the other contains Units. Or, it is the finding what will be the Sum of any Number added to itself, or repeated, as often as there are Units in another; being a compendious Kind of Addition.

2. What is to be observ'd in order to understand this Rule?

A. Take particular Notice, that of the two given Numbers the one is call'd the *Multiplicand*, which is the Number to be multiplied; the other the *Multiplier*, by which we multiply; and the third Number, arising from the Operation, is call'd the *Product*, which is the *Multiplicand* so often added to itself as there are Units in the *Multiplier*.—But before any Progress can be made in Multiplication, it is necessary for the Learner to get the following Table by heart, and that very perfectly.

E. 2

MUL.

MULTIPLICATION TABLE.

MULTIPLICATION TABLE

3 times	{	3 is	9
		4	12
		5	15
		6	18
		7	21
		8	24
	{	9	27

4 times	{	4	16
		5	20
		6	24
		7	28
		8	32
		9	36

5 times	{	5	25
		6	30
		7	35
		8	40
		9	45

6 times	{	6 is	36
		7	42
		8	48
	{	9	54

7 times	{	7	49
		8	56
		9	63

8 times	{	8	64
		9	72

9 times	{	9	81

12 times	{	2	24
		3	36
		4	48
		5	60
		6	72
		7	84
		8	96
		9	108

Wh

Wh

When this Table is perfectly learn'd by heart, so as to know the Product of any two Figures multiplied one by another without Hesitation, you may proceed to work any Sum proposed.

Q. How are Numbers to be set down in order for Multiplication.

A. First set down the *Multiplicand*, which is generally the greater of the given Numbers, and then the *Multiplier* underneath it, Units under Units, Tens under Tens, &c.

Q. How do you proceed in the Operation?

A. Having drawn a Line under the Multiplier, begin with the Figure in the Units Place, and with that multiply the Figure in the same Place of the Multiplicand, setting down the Product, if it be less than 10, under the Line in the Place of Units; but if the Product be more than 10, (or any Number of Tens) set down the

E 3

Overplus,

Overplus, as in Addition, and carry the Ten or Tens in Mind till you have multiplied the next Figure of the Multiplicand with the same Figure of the Multiplier, and to their Product add 1 for every 10 in the former Product, setting down the Overplus above the Tens as before; and so proceed till you come to the last Figure of the Multiplicand, where the whole Product must be set down, because the Tens can be carried no farther.—But an Example will make this plainer.

Let it be required to multiply 1745 by 6, and the Work will stand thus:

Multiply	1745	<i>Multiplicand.</i>
by	6	<i>Multiplier.</i>

10470 *Product.*

Here I begin with the Units, as above directed, saying, 6 times 5 is

30

30, which being just 3 Tens and nothing under nor over, I set down 0, and carry 3 to the next Place, saying, 6 times 4 is 24 and 3 is 27, which being 7 more than 2 Tens or 20, I put down 7 and carry 2. Then I say, 6 times 7 is 42, and 2 is 44, which Number containing 4 Tens and 4 over, I set down 4 and carry 4, saying, 6 times 1 is 6 and 4 is 10, which I set down, having gone through all the Figures of the Multiplicand, and the Product appears to be 10470, as you see above.

Q. But how do you proceed when there are more than one Figure in the Multiplier?

A. When the Multiplier consists of two or more Figures, begin with that in the Units Place, as before; and having gone through the Multiplier, and set down the Product under the Line, do the same by the Fi-

E 4

gure

gure in the Tens Place, setting down this Product one Remove nearer to the Left Hand, that is, its first Figure under the second of the former Product, the second under the third, &c. The same is to be observ'd in setting down the Product of each Figure of the Multiplier; and then drawing a Line under the several Products, and adding them together, you have the general Product required.

Example.

Multiply 1745
by 36

10470
5235

Gen. Product 62820

Here the Product by 6 being found and set down as in the former Example,

ample, I proceed to multiply by 3, saying, 3 times 5 is 15, that is, 5 more than 10, therefore I set down 5, the first Figure of this Product, under 7, which stands in the second Place of the former Product, as above directed. Then I say, 3 times 4 is 12 and 1 that I carry is 13, set down 3 and carry 1; then, 3 times 7 is 21 and 1 is 22, set down 2 and carry 2; lastly, 3 times 1 is 3 and 2 is 5, which I put down, and the Multiplication is finish'd; and by adding the two Products together the general one is found to be as above express'd.— To make this plainer take another Example:

Mul.

[58]

Multiply 13326
by 4839

$$\begin{array}{r}
 119934 \\
 39978 \\
 106608 \\
 53304 \\
 \hline
 64484514
 \end{array}$$

Q. What is to be done when Cyphers are intermix'd with Figures in the *Multiplier*?

A. Proceeding with the Figures as above directed, when you come to a Cypher in the Multiplier, instead of making a Line of Cyphers among the other Products, set down one Cypher directly under that in the Multiplier, and begin the Multiplication again with the next Figure, putting down the Product in the same Line with the Cypher: But observe, that the

the first Figure, or Cypher, of the next Product (if such there be) must be set down a Degree farther towards the Left Hand, not immediately under the Figure that stands next to the Cypher above; as may be understood by the second of the following Examples.

Example 1.

$$\begin{array}{r}
 4392 \\
 403 \\
 \hline
 13176 \\
 175680 \\
 \hline
 1769976 \\
 \hline
 \end{array}$$

Example 2.

$$\begin{array}{r}
 62725 \\
 2307 \\
 \hline
 439075 \\
 1871750 \\
 125450 \\
 \hline
 144606575 \\
 \hline
 \end{array}$$

N. B. It will be the same thing, when you come to a Cypher in the Multiplier, if you leave its Place vacant in the Product, or two Places if there be two Cyphers together, &c. pro-

proceeding to multiply by the next significant Figure, and always remember to put down the first Figure or Cypher of a Product perpendicularly under the Figure by which you multiply.

Q. Is there not a short Way of multiplying when you have Cyphers on the Right Hand of the given Numbers ?

A. Yes; when either your Multiplicand, or Multiplier, or both, have one or more Cyphers to the Right Hand, only multiply by the significant Figures, and then put the Cyphers contain'd in both or either of them to the Right Hand of the Product; as in the following Examples.

200	232	4000
3	200	3000
600	46400	12000000

000
000
000

In the two first Examples the Multiplicand and Multiplier are not set down according to the general Rule, that the Work may appear the plainer; but if they had been placed as usual, the Product would have been the same.

N. B. When you are to multiply any Number by 10, 100, 1000, &c. 'tis only annexing one, two, three, or more Cyphers to the Multiplicand, and the Work is done. Suppose, for Instance, I would multiply 275 by 10, the Addition of one Cypher gives me the Product, *viz.* 2750; if I multiply by 100, I add two Cyphers, and it makes 27500, &c.

There is also a short Way of multiplying by 5, namely, to add a Cypher to the Multiplicand and then halve it, which Half is the Product. For Example, if I would multiply 244 by 5, by annexing a Cypher it becomes

becomes 2440, the Half whereof is 1220, which is the Product required.

2. How do you multiply Numbers of divers Denominations?

A. Begin with the least, and so proceed from one to another till you come to the greatest, carrying from the lower Denomination to the higher, as you do in Addition.—Let us first give an Example in the Multiplication of Money.

Suppose I have three Purfes, and 9*l.* 3*s.* 7*d.* in each of them, how much does the Whole amount to?

This might be known by Addition, that is, by setting down the given Sum 3 times, and then adding them all together; but all such Questions as these are much more expeditiously answer'd by Multiplication. I therefore put down the Sum as follows:

Mul

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Multiply	9	— 3	— 7
by			3
Answer	27	— 10	— 9

Here I say, 3 times 7 is 21, which being 9 more than 12, that is Nine-pence above a Shilling, I set down 9 under the Pence, and carry 1 to the Shillings, saying, 3 times 3 is 9 and is 10, which being less than a Pound I have therefore none to carry, but set down 10 under the Shillings; then I say, 3 times 9 is 27, which I put down under the Pounds, and the Work is compleated.

Again: If 1 Piece of Cloth cost $l. 9s. 6d. \frac{1}{2}$, what will be the Price of 8 Pieces?

Multiply

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Multiply	7	9	$6\frac{1}{2}$
by			8
Answ.	59	16	4

In this Example I begin with the Halfpenny, saying, 8 Half-pence make 4 Pence, which I carry to the Pence and say, 8 times 6 is 48 and is 52; which being 4 Pence above Shillings, I set down 4 under the Pence, and carry 4 to the Shillings saying, 8 times 9 is 72 and 4 is 76 which is equal to 3 Pounds 16 Shillings; I therefore put down 16, and carry 3 to the Pounds, saying, 8 times 7 is 56 and 3 is 59; which being set down, as above, we have the Answer to the Question.

2. How do you proceed when the Multiplier consists of two Figures?

A.

A. If the Multiplier be above 12, the Question is sometimes answer'd by two Multiplications, and sometimes by three, and adding the Product of the last to the Product resulting from the two former.---Suppose it be ask'd, *What 15 Yards of Lace come to at* 1. 3s. 6d. *per Yard?* Here I consider, that as 3 times 5 make 15, I can perform this Work in two Multiplications, viz. by 3 and 5; as in the following Example.

	<i>l.</i>	<i>s.</i>	<i>d.</i>	
Multiply	1	— 3	— 6	
by			3	
Multiply	3	— 10	— 6	Pr. of 3.
by			5	
Answer	17	— 12	— 6	Pr. of 15.

This Example, I apprehend, needs
F no

no particular Explanation; for the given Price being first multiplied by 3, and then the Product of that by 5, the Result of these two Multiplications is plainly the Answer required.— But if it be ask'd, *What is the Product of 2l. 13s. 4d. multiplied by 39?* This Question cannot be answer'd by two Multiplications like the former, because there are no two Numbers which multiplied together will produce 39 exactly. Endeavouring however to come as near to it as possible; I consider that 4 times 9 make 36, and therefore I make use of 4 and 9 as Multipliers in this Operation. Having thus found the Amount of the given Sum multiplied by 36, I multiply it by 3 which makes 39, and the two Products added together give the Answer to the Question; as you may understand by a View of the Work.

Mult.

$$\begin{array}{r} \text{Mult.} \quad \text{2} - 13 - 4 \\ \text{by} \quad \quad \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} \text{And} \quad 10 - 13 - 4 \text{ Pr. of 4.} \\ \text{by} \quad \quad \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Add} \quad \left\{ \begin{array}{l} 96 - 00 - 0 \text{ Pr. of 36.} \\ 8 - 00 - 0 \text{ Pr. of 3.} \end{array} \right. \\ \hline \end{array}$$

$$\begin{array}{r} \text{Answ.} \quad 104 - 00 - 0 \\ \hline \end{array}$$

I forbear giving more Examples of this Nature, for if these be well understood it will be no great Difficulty to multiply by any Number whatsoever, observing the same Method.— But it may not be amiss to set down another Example, in order to shew the Scholar one considerable Use of Multiplication, namely, its bringing great Denominations into small, as Pounds into Shillings, Pence, or Farthings.

F 2

Suppose,

Suppose, for Instance, I would know how many Shillings there are in 36*l*. I need only multiply 36 by 20, (because 20*s*. make 1 Pound) and the Product gives the Number of Shillings required.

$$\begin{array}{r} \text{Multiply } 36 \\ \text{by } 20 \\ \hline \end{array}$$

Prod. 720 Shillings in 36*l*.

Now if 720 were multiplied by 12, (there being so many Pence in a Shilling) the Product would give the Number of Pence; and if that Product were again multiplied by 4, (the Farthings in a Penny) it would shew the Number of Farthings contain'd in 36 Pounds.—But more of this when we come to speak of *Reduction*.

Q. In what Manner is *Multiplication* to be proved?

A.

A. There are several Ways of proving it, first by Division, for when the Product divided by the Multiplier quotes the Multiplicand, or divided by the Multiplicand quotes the Multiplier, the Work is certainly right: But the Scholar having not yet learnt Division, the bare mention of this is sufficient. Another Way is to cast the Nines out of the Multiplicand, Multiplier, and Product; but this Method being erroneous, we recommend the following, *viz.* Make that which was your Multiplicand your Multiplier, and that which was your Multiplier your Multiplicand, and the Product will be the same if the Work be truly perform'd, as will appear by setting down one Example.

F 3-

Multiply

[70]

Multiply 243
by 176

1458
1701
243

42768

176
243

528
704
352

42768

CHAP. V.

Of DIVISION?

Q. WHAT is DIVISION.

A. It is a Rule whereby we discover how often one Number is contain'd in another. As Multiplication is a compendious Way of Addition, so Division supplies the Place of many Subtractions.

Q. What are the Terms made use of in this Rule?

A.

A. They are four, namely, the *Dividend*, or Number to be divided; the *Divisor*, or Number by which we divide; the *Quotient*, or Number arising from the Division, which shews how often the Divisor is contain'd in the Dividend; and lastly the *Remainder*, which is the Number left out of the Dividend after the Divisor has been taken from it as often as is express'd in the Quotient; but this is merely accidental, for sometimes there is no Remainder at all.—*N. B.* The Remainder must always be less than the Divisor.

Q. Are there not two Sorts of Division?

A. Yes, *Single* and *Compound*.

Q. What is *Single* Division?

A. It is call'd *Single* when the Divisor is but one Figure, and the Dividend but two at most.—Any Question of this Kind is answer'd by the

Multiplication-Table; as if 63 were to be divided by 7, the Answer will be 9. Here 63 is the *Dividend*, 7 the *Divisor*, and 9 the *Quotient*, shewing that the Number 7 is contain'd 9 times in the Number 63.

Q. What is *Compound* Division?

A. It is call'd *Compound* when the Dividend consists of more Figures or Cyphers than two, and the Divisor of one or more Figures or Cyphers.

Q. How do you set down a Sum in order to be divided?

A. First write down your Dividend, suppose it be 365, and on the Left Hand of it your Divisor, suppose 7, separating them by a crooked Line, or *Parenthesis*; then make another crooked Line on the Right of the Dividend, where you must place your Quotient when found; and the Work will stand as follows:

David,

*Divid.**Divisor* 7) 365 (

Q. How do you proceed in the Operation?

A. As I cannot take 7 out of 3, the first Figure of the Dividend, I therefore reckon the first and second together, and consider how often I can take 7 out of 36; which being 5 times, I set down 5 in the Place appointed for the Quotient, and with that I multiply the Divisor 7, saying 5 times 7 is 35, which I put down exactly under 36 the two first Figures of the Dividend: Then drawing a Line I subtract 35 from 36, and there remains 1, to which bring down the last Figure of the Dividend, viz. 5, and then there is 15 for a new Dividend, or Dividual, to work upon. I therefore consider again, how often 7 may be had out of 15, which being twice, I place 2
in

in the Quotient next to the 5, and again multiply the Divisor by 2, saying, twice 7 is 14, which I set down under the 15, from whence I subtract it, and find 1 to be the Remainder; as will be better understood by a View of the Work at length.

Divid.

Divisor 7) 365 (*52 Quotient.*

35

15

14

1

Remainder.

Thus I find that the Number 7 is contain'd 52 times in 365, besides which there is 1 remaining: And this Operation is the same as if I had ask'd how many Weeks there are in a Year; for there being 365 Days in a Year, and 7 Days in a Week, the Number 365

divided

divided by 7 shews that there are 52 Weeks, and 1 Day over.

N. B. If there had been more Figures or Cyphers in the Dividend, they must all have been brought down, one by one, (and never more than one at a time) and placed with the remaining Number after Subtraction, as the 5 was brought down and added to the 1 in the Example above. You are also to observe, that for every Figure or Cypher brought down from the Dividend in order for a new Operation, there must be a Figure or a Cypher set down in the Quotient. And that you may the better remember what Figures or Cyphers have been so brought down and done with, you may make a Dot under them in the Dividend, as you see in the following Example.

Suppose a Father dies, and leaves 8440 $\frac{1}{2}$ to be equally divided amongst 8 Chil-

[76]

8 Children, how much will be the Share of each?

8) 8440 (1055

8...

—

4

0

—

44

40

—

40

40

—

0

—

0

—

Here I begin by enquiring how many times the Divisor 8 can be had out of 8 the first Figure in the Dividend, which being but once, I set down 1 in the Quotient, and 8 under 8 in the Dividend, from whence I subtract it and there remains 0; but

Cypher

Cypher being of no Value on the Left Hand of a Figure, I do not put it down, but leave the Place vacant. Then bringing down the 4 from the Dividend, and making a Dot under it to shew that I have done so, I say, 8 from 4 cannot be had, therefore I put a Cypher in the Quotient; and as I cannot multiply the Divisor by Nothing, I also put down 0 under the 4, and proceed to Subtraction, saying, 0 from 4 and there remains 4; to which I bring down the next 4 in the Dividend, making a Dot under it as before, and this gives me a new Dividual, viz. 44. I then consider that 8 may be had 5 times out of 44, and accordingly put down 5 in Quotient; then multiplying 8 by 5, which makes 40, I place 40 under the 44, and subtract it as usual: To the remaining 4 I then bring down the 0 from the Dividend, which making 40 I consider

sider again how many Eights that Number contains, and finding them to be 5 I set down another 5 in the Quotient, by which I likewise multiply the Divisor, saying, 5 times 8 is 40; and this being put down and subtracted from the other 40, the Work is compleated, and nothing remains. The Quotient therefore, 1055, gives the Answer to the Question, for just that Number of Pounds is the Share of each Child.

Q. Is there no other Method of working in Division than this?

A. Yes; when the Divisor consists only of one Figure, as in the Examples above, the Work may be perform'd in a much more compendious Manner, and which in my Opinion will be more easy to the Learner. In this Method you are to draw a Line under the Dividend, and set down under its first Figure how often the Divisor

divisor is contain'd in it; if any thing remains, imagine it placed before the next Figure, and consider how often the Divisor is contain'd in the Sum it makes; set down the Number underneath, as before; and so proceeding through all the Figures of the Dividend, put down what remains at last where the Quotient stands in the former Examples.—An Instance or two will make this plain.

Let it be required to divide 78906 by 4, that is, into four equal Parts; the Work, when finish'd, will stand thus:

	<i>Dividend</i>
<i>Divisor</i>	4) 78906 (2 <i>Remainder.</i>

<i>Quotient</i>	19726
-----------------	-------

In working this Example I say, 4 may be had once in 7, and there remains 3; I therefore set down 1 under

der the Line, and suppose the 3 to be placed before 8; the next Figure in the Dividend, which makes it 38. then proceed to enquire how many times 4 in 38, which being 9 times and 2 over, I set down 9 in the Quotient, and carry 2 to the 9 in the Dividend, which makes it 29; the the Fours in 29 are 7, and 1 over therefore I put down 7, and supposing the 1 to be placed before the 0 in the Dividend, which would make 10, I proceed to consider how many times 4 is contain'd in that Number which being twice, and 2 over, I set down 2 in the Quotient, and carrying the Overplus 2 to the 6 in the Dividend I call it 26; then lastly enquire how many times 4 is contain'd in 26, which being 6 times and 2 over, I put down 6 under the Line, and the remaining 2 on the Right Hand of the Dividend, and the Operation

Operation is compleated.—I shall set down one more Example for the Learner's Practice.

$$6)479620(4$$

$$\underline{079936}$$

N. B. This is the shortest Way of Division that can be by a single Figure; and as it is a Halving, Thirding, Fourthing, &c. of the Dividend, we chuse to express the Divisor in the Manner of a Fraction, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. as in the following Examples:

$$\begin{array}{r} 23412 \\ \frac{1}{3})\text{---} \\ 7804 \end{array}$$

$$\begin{array}{r} 42721 \\ \frac{1}{2})\text{---} \\ 6103 \end{array}$$

In this Case the Method of working is the same as above, saying, the third of 23 is 7 and 2 over, which annex'd to the 4 makes 24; then

G

the

the Third of 24 is 8; the Third of 1 cannot be had, therefore put down a Cypher; but the 1 reckon'd with the 2 makes 12, the Third of which is 4, without any Remainder, therefore set down 4 in the Quotient, and the Work is finish'd.—And thus Division by any single Figure is readily perform'd.

2. But how do you proceed when the Divisor consists of two or more Figures?

A. The working in this Sort of Division is not quite so easy, but I shall endeavour to make it as plain as the Nature of the Thing will admit.—Suppose then that 352 Oranges were to be equally divided among 32 Boys, how many would each Boy have to his Share?

Here 352 is the Dividend, and 32 the Divisor, which I set down in the usual Manner, and having gone th

the Operation as explain'd below, the whole Work stands as follows;

$$32 \overline{) 352} (11$$

$$32$$

$$\hline 32$$

$$32$$

$$\hline$$

$$0$$

$$\hline$$

First I enquire how many times 32 is contain'd in 35, the two first Figures of the Dividend, which I find to be once; and it would have been the same thing if I had ask'd how often 3, the first Figure of the Divisor, was contain'd in 3 the first of the Dividend: I therefore put down 1 in the Place of the Quotient, saying, once 32 is 32, which I place under 35 in the Dividend; and subtracting it from thence there remains 3: To

G 2

this

this 3 I bring down 2 from the Dividend, and these give me a new Dividual, namely 32, which being the same as the Divisor must necessarily contain it once and no more; I therefore set down another 1 in the Quotient, and also 32 under 32 the last Dividual, from whence I subtract it, and nothing remains.—And here the Quotient shews how many Oranges each Boy must have, namely *eleven*.

Let us make another Trial, and divide 1231 by 36, which is the same thing as if I should ask how many Yards there are in 1231 Inches.

Having set down the Divisor and Dividend, $36 \overline{)1231}$, I am sensible at first Sight that the two Figures of which the Divisor consists cannot be taken from the two first Figures of the Dividend, and therefore I must have three, and enquire how often 36 may be had in 123; but as this is

not

not readily answer'd by a young Practitioner in Division, I shall go an easier Way to work, (as may always be done where the Divisor consists of several Figures) and try how often the first Figure of the Divisor may be had in the first of the Dividend; but as in this Example it cannot be had at all, I take two Figures of the Dividend, and ask how many times 3 in 12. The Answer is plainly 4 times; but before I set down 4 in the Quotient, I multiply the Divisor by it, (either in my own Mind, or upon some waste Piece of Paper) to see whether the Product will not amount to more than the three first Figures of the Dividend, viz. 123, from whence it is to be subtracted; and finding that 4 times 36 is 144, which cannot be taken from 123, I only put down 3 in the Quotient, and then multiply the Divisor by it, placing the Product, viz.

G 3

108,

[86]

108, under 123 in the Dividend, and after Subtraction there remains 15, the Work standing thus:

$$\begin{array}{r} 36) 1231 (3 \\ \underline{108} \\ 15 \end{array}$$

To this 15, I bring down 1, the next and last Figure of the Dividend, which gives me 151 for a new Dividual. Then, instead of asking how many times 36 in 151, I enquire (as before) how often 3 in 15, which I readily see to be 5 times; but multiplying the Divisor by 5, I find the Product would again be too large for Subtraction, and therefore I set down but 4 in the Quotient, by which I multiply the Divisor, and the Product is 144. This being placed under the Dividual 151, and subtracted from it, I find 7 remaining; and there being

being no more Figures to bring down to it from the Dividend, the Work is finish'd, and appears as follows:

$$36)1231(34\frac{7}{36}$$

108

151

144

7

Hence I learn, that in 1231 Inches there are 34 Yards, and 7 Inches, or 7 Parts of a Yard out of 36; for the Remainder is always a Fraction, and is frequently placed over the Divisor on the Right Hand of the Quotient, as in the Example above.

In the same Manner any larger Number may be divided; but when the Divisor consists of many Figures, it is a great Ease to the Memory to make a Table of it, multiplying it

severally by all the nine Digits, and setting down the Numbers in the following Order.—Let 642 be the Divisor, which must be placed over-against the 1; then multiply 642 by 2, and opposite to it set down the Product; then multiply 642 by 3, placing the Product over-against it, and so proceed till you have finish'd the Table.

1	642
2	1284
3	1926
4	2568
5	3210
6	3852
7	4494
8	5136
9	5778

Now suppose it were required to divide 146728 by 642, I set down the Divisor and Dividend as usual, 642)146728, and consider out of how many of the foremost Figures or Places
of

of the Dividend I must first take the Divisor. It is plain at first Sight that three will not do, therefore I take four, *viz.* 1467; putting a Dot under the 7, to shew that the Divisor reaches so far into the Dividend. Then asking how often 642 is contain'd in 1467, I look in the Table, but not finding that Number exactly, I look for the next less, which is 1284; and this standing over-against the Number 2, I thereby know that the Divisor 642 is twice contain'd in the Dividend 1467. I therefore put down 2 in the Quotient, and subtracting 1284 from 1467 I find 183 remaining, which being set down the Work stands thus:

$$642)146728(2$$

$$\begin{array}{r} 1284 \\ \hline \end{array}$$

$$183$$

Proceed:

Proceeding as usual, I now make a Dot under 2, the next Figure in the Dividend, and bring it down to the Remainder 183, which gives me 1832 for a new Dividual. Then enquiring how often 642 is contain'd in 1832, I find by the Table it is no more than twice, for 1284 is again the next less Number ; I therefore put down 2 in the Quotient, and subtracting 1284 from the Dividual 1832, there remains 548, the Work appearing thus:

$$642) 146728(22$$

$$\underline{1284}$$

$$1832$$

$$\underline{1284}$$

$$548$$

I then

I then make a Dot under 8, the last Figure in the Dividend, and bring it down to the remaining 548, whereby I have another Dividend to work upon, viz. 5488, and therefore I enquire how often 642 can be had in that Number, which I find by the Table is 8 times; for 5136, the next less Number to 5488, stands opposite to 8, which being placed in the Quotient, and 5136 subtracted from 5488, the Remainder is 352; and there being no more Figures to bring down from the Dividend, the Work is finish'd, and stands as follows:

642

$$642 \overline{) 146728} \quad (228 \overline{) 146728} \quad \begin{array}{r} 228 \\ 486 \\ 144 \\ 28 \\ 28 \end{array}$$

$$1284$$

$$1832$$

$$1284$$

$$5488$$

$$5136$$

$$352$$

This Method of making a Table of the Divisor will be of great Service to young Scholars, till they are well practised in Division; and by beginning with small Sums, and so proceeding to greater, they will soon acquire a sufficient Knowledge in this Branch of Arithmetic. I shall set down one Example more with the same Divisor, without a verbal Explanation of the Work,

Work, that the Learner may try how he can make use of the Table. Divide 2579683 by 642, and the Work when finish'd will stand thus:

$$642 \overline{) 2579683} \quad (4018 \frac{127}{642}$$

$$\underline{2568}$$

$$\underline{1168}$$

$$\underline{642}$$

$$\underline{5263}$$

$$\underline{5136}$$

$$\underline{127}$$

Q. Is there not sometimes a short Way of working in *Division*, as well as in *Multiplication*?

A. Yes; I have already spoken of such a Method when the Divisor is only a single Figure, and in some Cases

ses the Work may be shorten'd, even when the Divisor consists of several Places.

First, When the Divisor has one, two, or more Cyphers to the Right Hand, separate them from the significant Figures by a Comma; and in like manner separate the same Number of Figures or Cyphers from the Right Hand of the Dividend; then divide the remaining Figures of the Dividend towards the Left Hand by the significant Figures of the Divisor, and if there be any Remainder, bring down to it the Figures cut off from the Dividend, for they are always a Part of the Remainder, and sometimes the Whole: As in the following Examples.

Example

Example 1.

$$\begin{array}{r} 24,00 \overline{) 482,20} \quad (20 \overline{) 220} \\ \underline{48} \end{array}$$

220 Rem.

Example 2.

$$\begin{array}{r} 3,000 \overline{) 169,242} \quad (56 \overline{) 1242} \\ \underline{15} \end{array}$$

19

18

1242 Rem.

Secondly, It appears from the foregoing Rule, that when we are to divide by 10, 100, 1000, &c. we have nothing more to do than to separate so many Figures or Cyphers from the Right of the Dividend as there are Cyphers in the Divisor; and the Figures on the Left Hand of the Mark of

of Separation are the *Quotient*, as those on the Right are the *Remainder*.—Suppose, for Example, the Number 31467 were to be divided by 10, 100, and 1000, the Work and the Answer appear at one View as follows:

10) 3146,7

100) 3,14,67

$$1000 \overline{) 31,467}$$

In the first Instance 3146 is the Quo-
tient, and 7 the Remainder; in the
second the Quotient is 314, and the
Remainder 67; and in the last the
Quotient is 31, and 467 the Remain-
der.

2. How do you divide Numbers of divers Denominations?

A. Suppose 65*l.* 17*s.* 6*d.* were to be divided amongst 5 Men, set down the Sum as follows:

5) 65 : 17 : 6 (

Here

Here I consider how many times 5 may be had in 6, the first Figure of the Pounds, which being once, I set down 1 in the Place of the Quotient, and carry the overplus 1 to the 5, the next Figure in the Pounds, which makes it 15. Then I say, 5 in 15 may be had 3 times, and therefore I put down 3 in the Quotient. Having done with the Pounds I proceed to the Shillings, saying, the Fives in 17 are 3 and 2 over; I therefore set down 3 in the Quotient, and considering that the overplus 2 are 2 Shillings, or 24 Pence, I add 24 to the 6 in the Place of Pence, which makes them 30 in all. Then I say, 5 out of 30 may be had 6 times, and nothing remains; therefore I put down 6 in the Quotient, and so the Work being finish'd stands thus:

$$\begin{array}{r} \text{.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \\ 5) 65 : 17 : 6 \quad (13 : 3 : 6 \end{array}$$

For another Example, let it be required to divide 4*l.* 15*s.* 6*d.* among 6 Men, and the Work when completed will appear as follows:

$$\begin{array}{r} \text{.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \\ 6) 4 : 15 : 6 \quad (0 : 15 : 11 \end{array}$$

In this Instance, as I cannot take the Divisor 6 from the 4 Pounds in the Dividend, I put down a Cypher in the Quotient, and considering how many Shillings are contain'd in 4 Pounds, *viz.* 80, I add 80 to the 15 in the Place of Shillings, which makes 95 in all. Then I enquire how many times 6 in 95, which being 15 times and 5 over, I set down 15 in the Quotient; and the overplus 5 Shillings containing 60 Pence, I add 60 to the following 6, making 66 in all; in
which

which Number the Divisor being contain'd just 11 times without any Remainder, I set down 11 in the Quotient, and the Operation is finish'd, shewing 15s. 11d. to be each Man's Share of the Money.

Q. In what Manner is *Division* to be proved?

A. Multiplication and Division mutually prove each other: For as in Multiplication, if you divide the Product by the Multiplier, the Quotient will be the Multiplicand; so in Division, if you multiply the Quotient by the Divisor, (taking in the Remainder if there be any) the Product will be the Dividend, or else the Work is not rightly perform'd.

To make this plain by an Example, let us divide 8280 by 24, and the Work will appear as follows:

H 2

24)

[100]

24) 8280 (345

72

108

96

120

120

0

In order to prove this according to the Rule laid down, we must multiply the Quotient 345 by the Divisor 24, and the Product will be the Dividend 8280, as appears below:

345

24

1380

690

8280

N.

N. B. Division may also be proved by Division; that is, if you divide the Dividend by the Quotient, the Quotient will be your former Divisor: As in the Example above, if 345 were made the Divisor, the Quotient would be 24.

CHAP. VI.

Of REDUCTION.

2. **W**HAT other Rules are there in *Arithmetic*?

A. We have now gone through the five *fundamental* Rules, upon which all the succeeding ones depend, and are come to that call'd REDUCTION, which is usually taught immediately after *Division*.

2. What is *Reduction*?

A. It is an Application of Multiplication and Division, shewing how to reduce Numbers of one Denomination to another, still retaining the same

same Value, though in different Terms.

—*First*, All great Names are brought into small by multiplying by so many of the little ones as make one of the great. Thus any Number of Pounds multiplied by 20 are reduced into Shillings, because 20 Shillings make 1 Pound: Shillings multiplied by 12 are reduced into Pence, and these multiplied by 4 are reduced into Farthings, as has been observ'd already in the Chapter of *Multiplication*. —*Secondly*, Small Names are brought into greater by Division; as Farthings into Pence by being divided by 4, Pence into Shillings by 12, and Shillings into Pounds by 20: For Multiplication and Division are too contrary Operations. —*Thirdly*, In changing one Sort of Money, Weight, Measure, &c. into another, both must be reduced into the same Denomination, and the one divided by the other.

Q. Give me an Example of the first Sort of *Reduction*.

A. Let us begin with Money, and ask *how many Shillings, Pence, and Farthings there are in 27 Pounds*. To know this you are to multiply the Pounds by 20, the Shillings by 12, and the Pence by 4, and the Work will stand as follows:

$$\begin{array}{r} 27\text{l.} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 540 \text{ Shillings.} \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 1080 \\ 540 \\ \hline \end{array}$$

$$\begin{array}{r} 6480 \text{ Pence.} \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 25920 \text{ Farthings.} \\ \hline \end{array}$$

H 4

N. B.

N. B. Pounds may be brought into Pence at once by multiplying by 240, or into Farthings by multiplying by 960, there being so many Pence and Farthings in a Pound.

One Example more of this Kind will be sufficient; as, *How many Farthings are there in 12l. 7s. 6d.?*—
The Work when finish'd appears thus:

l.	s.	d.
12	7	6
20		
247 Shillings.		
12		
500		
247		
2970 Pence.		
4		
11880 Farthings.		

Here

Here the Manner of working is the same as in the first Example, only in multiplying the Pounds we take in the Shillings, to which Name the Pounds are reduced by that Multiplication; and in multiplying the Shillings we take in the 6 Pence, that being the Name to which the Shillings are reduced.

A few more Examples in *Weights and Measures* will make this Rule very easy and intelligible to the young Scholar.

In 21 C. wt. how many Quarters and Pounds?—Here you are to consider, that as 4 Quarters make a Hundred weight, multiplying the Hundreds by 4 will reduce them to Quarters; and as 28 Pounds make a Quarter, multiplying them by 28 will bring them into Pounds, and give an Answer to the Question, as follows:

21 C. wt.

[106]

21 Cwt.

4

—

84 Quarters.

28

—

672

168

—

2352 Pounds.

—

If the Pounds were to be multiplied by 16, the Product would be the Number of Ounces; and if the Ounces were also multiplied by 16, the Product would give the Number of Drams contained in 21 Hundred Weight.

In 123 Hogsheads how many Gallons and Pints?—Here you are to remember (or consult your Tables) that 6 Gallons make a Hogshead, by which Number

umber you are therefore to multi-
y to reduce the Hogsheads to Gal-
ns; then multiply the Gallons by 8
reduce them to Pints, and the Work
ill appear as follows:

$$\begin{array}{r} 123 \text{ Hogsheads.} \\ 63 \\ \hline 369 \\ 738 \\ \hline \end{array}$$

$$\begin{array}{r} 7749 \text{ Gallons.} \\ 8 \\ \hline 61992 \text{ Pints.} \\ \hline \end{array}$$

In 546 Yards how many Feet and
ches?—Multiply the Yards by 3 to
ring them into Feet, and the Feet
y 12 to bring them into Inches; and
e Work will stand thus:

546 Yards.

[108]

546 Yards.

3

1638 Feet.

12

3276

1638

19656 Inches.

In 12 Weeks how many Days, Hours and Minutes? — Multiply the Weeks by 7 to reduce them to Days, Days by 24 to reduce them to Hours and the Hours by 60 to bring them to Minutes.

12 Weeks

[109]

12 Weeks,

7

84 Days.

24

336

168

2016 Hours.

60

120960 Minutes.

These Examples may suffice for this
 kind of Reduction, which is call'd
 Reduction *Descending*, because it brings
 great Denominations into smaller.
 The next Branch of this Rule, which
 perform'd by Division, is call'd Re-
 duction *Ascending*, because it brings
 lesser Denominations into greater.

2. What Examples have you of
 this

this Sort of Reduction?

A. The foregoing Questions reversed will be so many Examples in this Branch of Reduction, and each Operation will prove the Truth of the other. First then,

How many Pence, Shillings, and Pounds in 25920 Farthings? In this Case, as I before multiplied by 20, 12, and 4, I must now divide by 4, 12, and 20.

4) 25920 Farthings.

12) 6480 Pence.

20) 540 Shillings.

27 Pounds.

Again: *How many Pounds in 1188 Farthings?*—Here, dividing as in the last Example, the whole Work will appear as follows:

4) 11880 Farthings.

12) 2976 Pence.

20) 247 Shillings and 6 Pence.

12 Pounds and 7 Shillings.

Thus I find that 11880 Farthings equal to 12*l.* 7*s.* 6*d.* And it could have amounted to the same if I had divided 11880 by 960, Number of Farthings in a Pound. Here it may be observ'd, that whatever odd Money, Weight, or Measure we take in as we multiply the several Denominations in *Reduction Descending*, the same will be a Remainder in *Reduction Ascending*; as the 7*s.* 6*d.* in the above Example.

In the next Place, *How many Quarter and Hundred-Weight are there in 2 Pounds?*—This is answer'd by dividing

[112]

dividing first by 28 the Number Pounds in a Quarter, and then by the Number of Quarters in a Hundred.

Pounds:

28) 2352 (84 Quarters

224

112

112

0

Here the Quotient shews the Number of Quarters to be 84, and divided by 4 shews the Number Hundreds, namely 21.—Thus the Examples prove the Truth of the corresponding ones in the former Part *Reduction*, as you will see by comparing them together.

I apprehend it is needless to run through more Examples of this Kind, whose already given being sufficient to shew the Nature of Reduction *Ascending* and *Descending*, so far at least as they are separately employ'd: But it is proper, by a few Examples, to explain the *Third Part* of this Rule, (if I may call it so) wherein *Multipli-*
cation and *Division* are both concern'd in answering the same Question.

Q. What is the Business of this Third Part of Reduction?

A. It is used in changing one Sort of Money, Weight, Measure, &c. into another; as Foreign Money into Sterling, or Sterling into Foreign.—
For Example, *In 459 French Crowns, each 4s. 6d. each, how many Pounds Sterling?*

This, and other Questions of the same Nature, may be answer'd several Ways, as the Scholar himself will find

find by a little Consideration and Practice: But the most obvious Method of proceeding in this Case is to multiply the Crowns by 9 to bring them into Sixpences, and to divide the Product by 40 to bring them into Pounds; as each Crown contains 9 Sixpences, and each Pound 40.—See the Work that follows:

$$\begin{array}{r}
 459 \text{ French Crowns,} \\
 \quad 9 \\
 \hline
 4,0) 413,1 \text{ (11 Sixpences remaining)} \\
 \hline
 103 \text{ Pounds.} \\
 \hline
 \end{array}$$

So that the Answer is 103*l.* 5*s.* 6*d.*
 —The Reverse of this, namely, How many French Crowns there are in 103*l.* 5*s.* 6*d.* Sterling, is found by multiplying 103 by 40, taking in the Sixpences, and then dividing the Product by 9; as is easy to be understood without farther Explanation.

[115]

In 524 Moidores, at 27 Shillings
 each, how many Guineas? Here mul-
 ty the Moidores by 27 to reduce
 them to Shillings, and then divide the
 product by 21, the Quotient will give
 the Answer required.

524 Moidores.

27

3668

1048

114148 (673 Guineas.

126

154

147

78

63

15 Shillings remaining.

I 2

This

This Work may be perform'd in shorter Manner, for when the Moidores are reduced to Shillings, you may divide them by 3, and then the Quotient by 7, and the last Quotient will be the Answer to the Question as appears below:

3) 14148 Shillings in 524 Moidores

7) 4716

673 Guineas, and 5 remaining

— that is, five 3's, or 15 Shillings.

To understand the Reason of this you must consider, that the Shillings being divided by 3 are thereby reduced to so many 3 Shillings as the Quotient expresses, and consequently these being divided by 7 are reduced to Guineas; because 7 times 3, or 21 times 7, make 21, the Number of Shillings in a Guinea. Observe a

that the 5 remaining are so many times Shillings, the Remainder being always of the same Name with the Dividend.

But there is still a more compendious Method of performing this Work, which is worth the Scholar's Notice, and the best that can be practised whenever the Case will admit of such Contraction. As 9 is one Third of 27, the Shillings in a Moidore, multiply the given Number of Moidores by 9, and the Product shews how many 3 Shillings they contain; which being divided by 7 are reduced to Guineas, as above; and so the whole Work appears in the following small Compass:

[118]

524 Moidores.

9

7) 4716 Three Shillings.

673 Guineas, 15s.

In 215 Portugal Pieces, of 36 Shillings each, how many Guineas? The Question may be answer'd in the same short Manner as the former; for as times 12 is 36, and 3 times 7 is 21, you need only multiply 215 by 12 and divide the Product by 7, and the Quotient is the Answer required.

215 Pieces.

12

7) 2580 Three Shillings.

368 Guineas, 12s.

In 246 Venetian Ducats, at 4s. 4d. each, how many Pounds Sterling? —
 To answer this, multiply the given Number of Ducats by 52, to bring them into Pence; then divide the Pence by 12 to bring them into Shillings, and these by 20 to bring them into Pounds; and the Work will appear as follows:

$$\begin{array}{r}
 246 \text{ Ducats.} \\
 52 \\
 \hline
 492 \\
 1230 \\
 \hline
 \end{array}$$

12) 12792 Pence.

2,0) 106,6 Shillings.

53 Pounds, 6 Shillings.

In 672 Ells Flemish, how many Ells English? — Here remember (as your
 I 4 Tables

Tables inform you) that the *Flemish* Ell is 3 Quarters of a Yard, and the *English* Ell 5 Quarters: Therefore multiply the given Number of *Flemish* Ells by 3 to reduce them to Quarters, then divide the Quarters by 5, and the Quotient will be the Answer to the Question, as appears from the Work at length:

672 Ells *Flemish*.

3

5) 2016 Quarters.

403 Ells *English*, 1 Quarter.

I forbear to give the Reverse of these Questions, as thinking the Manner of stating and working them will not be difficult to the young Scholar, but a proper Exercise for the Trial of his Ingenuity.

N. B.

N. B. There is a compendious Way of reducing Ells *Flemish* to Ells *English*, namely, to multiply them by 6 and cut off the Right-hand Figure of the Product, in the following Manner:

$$\begin{array}{r} 672 \text{ Ells } \textit{Flemish}. \\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Ells } \textit{English} \text{ } 403\frac{1}{2} \\ \hline \end{array}$$

In 456 Ells *English*, how many Yards—To answer this Question multiply by 5 to bring them into Quarters, and divide by 4 to reduce them to Yards, as follows:

$$456 \text{ Ells } \textit{English}.$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$4) 2280 \text{ Quarters.}$$

$$\begin{array}{r} 570 \text{ Yards.} \\ \hline \end{array}$$

N. B.

N. B. Yards are compendiously reduced to Ells by multiplying by 8 and cutting off the Right hand Figure or Cypher of the Product; thus:

$$\begin{array}{r}
 570 \text{ Yards.} \\
 \times 8 \\
 \hline
 4560 \\
 \hline
 \text{Ells}
 \end{array}$$

Enough has been said to explain the Nature of *Reduction*; I shall therefore conclude this Rule with one Question and its Answer, leaving the Operation as a Trial for the Learner.

In 722 French Livres at 20 d. each how many Pieces of Eight at 4s. 4d.
 — Answer, 277 Pieces and 3 Shillings over.

CHAP. VII.

Of the Rule of PROPORTION, or the Rule of THREE, Direct, Indirect, and Double.

Q. **W**HAT is the Use of this Rule?

A. It is of very great Use both in the Business of Common Life and in the Sciences, and is therefore, by way of Eminence, frequently call'd the *Golden Rule*.

Q. What is the Reason of its other Names?

A. It is call'd the *Rule of Three*, or *Rule of Proportion*, because by three Numbers given we find out a fourth, which bears such Proportion to the third, as the second bears to the first.

Q. How are Questions to be stated for working according to this Rule?

A. With respect to the stating any Question

Question observe, that of the three given Numbers two always contain a Supposition, and the third a Demand. The Number then on which the Demand lies, that is, to which the Interrogatives *What? How many? &c.* are annex'd, must always be set in the *third* Place, namely, on the Right Hand; and of the other two Numbers, that which is of the same Name or Kind with the Right hand Number must be set on the Left-Hand, or in the *first* Place; so that the remaining Number must of consequence stand in the *second*, that is, between them. Suppose, for Instance, it were ask'd, *If 3 Oranges cost 6 Pence, what will 24 Oranges cost?* the Numbers for working must be stated thus:

Or. Pence. Or.
If 3 — 6 — 24

Here you see that 24, the Number

of which the Question is ask'd, is placed on the Right-hand; the Number 3, which is of the same Kind, as it likewise denotes *Oranges*, is placed on the Left; and the Number 6, which is of a different Kind, as signifying *Pence*, is set in the Middle, according to the above Directions.

2. How is the Work to be perform'd?

A. Multiply the second and third Numbers together, then divide the Product by the first, and the Quotient will give the Answer to the Question, in the same Name with the middle Number, that is, in *Pence*; for you are to observe, that whatever Name the second Number bears at the Time of multiplying, the Quotient also bears when the Division is perform'd. — See the Work at length.

Or.

Or. Pence. Or.
If 3 — 6 — 24
6

3) 144 (

Ans. 48 Pence, or 4s.

This Question stated the contrary Way will serve for another Example, and prove the Truth of the former Operation.—Say then, *If 24 Oranges cost 48 Pence, what is the Price of 3 Oranges at the same Rate?*—In this Case the Numbers and Work according to the foregoing Directions will appear as follows;

Or.

Or. Pence. Or.
If 24 — 48 — 3
3

24) 144 (6 Answer.

144
—
0
—

18 3
6

72
4 326

Here the first and second Numbers multiplied together produce 144, which being divided by 24, the first Number, the Quotient is 6, that is, 6 Pence, the Price of 3 Oranges.—But for the better understanding this Rule, it is necessary to give some more Examples.

If 6 Gallons of Brandy cost 2l. 5s. what will 134 Gallons cost?—This is to be stated and work'd as follows:

134 Gall.
24
—
96
—
38

<i>Gall.</i>	<i>l.</i>	<i>s.</i>	<i>Gall.</i>
If 6	2	5	134
	20		45
	45		670
			536
			6) 6030
			2,0) 100,5

Anfw. 50*l.* 5*s.*

Here you see, the second or middle Number being of different Denominations, that is *Pounds* and *Shillings* is reduced to the lowest of those Names (*viz.* *Shillings*) by Multiplication, and this must always be done when it consists of several Names; for if it be *Pounds*, *Shillings*, and *Pence*, it must be reduced to *Pence*, that is, into the lowest

lowest Denomination mention'd.—

The Middle Number in this Example being thus reduced, the three Numbers for the Operation are 6 — 45 — 134: I therefore multiply the third Number by the second, that is 134 by 45, according to the Directions given, and the Product is 6030; which being divided by 6, the first Number, quotes 1005 Shillings, the Name of the middle Number 45; and these divided by 20 give 50*l.* 5*s.* for the Answer to the Question.

*If 4*s.* 6*d.* purchase 1 Yard and 1/2 of Linen, how many Yards will 12*l.* 12*s.* purchase?*

In this Question you are to observe, that the first and third Numbers are of different Denominations, the one being Shillings and Pence, and the other Pounds and Shillings: Now the first Business is to reduce these to the same Name, which must be the lowest
K mention'd;

mention'd; and this must always be done in the like Cases, immediately after the stating of the Question. Here then we are to bring the first and third Numbers into *Sixpences*; and the middle Number being also of different Names, that is, Yards and Half-yards, must be brought into the latter, agreeable to the Directions already given. When this is done, we are to multiply and divide according to the Rule and the Work will stand as follows:

<i>s.</i>	<i>d.</i>	<i>Yards.</i>	<i>l.</i>	<i>s.</i>
If 4	:	6	—	1 $\frac{1}{2}$ —
2	:	2	:	12 : 12

—	—	—
9	3	594

3

9) 1512

2) 168 Half-yards

Ans. 84 Yards.

Here

Here you see, the first Number is multiplied by 2 to reduce it to Sixpences, because 2 Sixpences make 1 Shilling; the second is also multiplied by 2 to reduce it to Half-yards, because 2 Half-yards make a Yard; and the third is brought into Sixpences by multiplying by 40, because 40 Sixpences make one Pound. This being done, the Numbers for working are 9—3—504; the third therefore being multiply'd by the second, the Product is 1512, and this being divided by the first Number, the Quotient is 168, which are Half-yards, the Name to which the middle Number was reduced; and these being divided by 2 the Quotient is 84 Yards, which is the Answer to the Question.

N. B. If there be any Remainder after you have multiplied your second and third Numbers together and divided by the first, such Remainder is

Part of an Unit of the Quotient, the Value of which is thus found. Multiply it by that Number of the next inferior Denomination which makes one of the Quotient, and divide the Product by the same Divisor you made use of in your first Division: Suppose, for instance, the Quotient be Pounds, multiply the Remainder by 20, and divide the Product by the first Divisor, and the Quotient will be Shillings; then if any thing remains, multiply it by 12, divide by the same Divisor, and the Quotient will be Pence; and if there be still a Remainder, multiply by 4, divide as before, and the Quotient will be Farthings.

Note also, When the first of the three given Numbers is an Unit, or One, you cannot divide by it according to the Rule, for 1 neither divides nor multiplies. In this Case, when you have multiplied the second and

third

third Numbers together, the Product is also the Quotient, giving the Answer to the Question in the Denomination of the middle Number, as appears in the following Example.

If 1 Pound of Silk cost 17s. what will be the Price of 264 Pounds at the same Rate ?

lb.	s.	lb.
If 1	— 17	— 264
		17
		1848
		264

Answer 4488 Shillings.

Here the Question is answer'd by Multiplication only, but the Answer being in Shillings is not readily understood till it be reduced to Pounds by dividing by 20, and then it appears to be 224*l.* 8*s.*

These Examples may suffice to shew the Manner of working all Questions in the *Direct* Rule of Three, that is, when the second and third Numbers are multiplied together, and the Product divided by the first: I shall now give a few Examples in the Rule of Three *Indirect* or *Inverse*, wherein the first and second Numbers are multiplied together, and their Product divided by the third.

Q. How are Questions stated in this Rule?

A. The same Way as in the *Direct* Rule; for the first and third Numbers must be of one Name, or so reduced; the Number of which the Question is ask'd must stand in the third Place; and the Number of a different Denomination in the Middle.

Q. How is the Answer to be found?

A. The Quotient, as before, will be the Answer, and of the same Name with

with the middle Number. For Example: If 9 Men would build a Wall in 15 Days, how many Men must be employ'd to build it in 5 Days?

In stating these Numbers observe the above Rule, and the Work will appear as follows:

	Days.	Men.	Days.
If	15	— 9 —	5
	9		

5) 135

Ans. 27 Men.

Here you see, the first Number being multiplied by the second, and the Product divided by the third, the Quotient is 27, the Answer to the Question. And you may also observe the *indirect Proportion* the Numbers bear to each other; for so much as the third is less than the first, so much is

the fourth greater than the second; and on the contrary, the fourth is always so much less than the second, as the third is greater than the first.— But let us have another Example.

If 60 Yards of Serge of 3 qrs. wide will line a certain Number of Coats, how many Yards will line the same Number when the Serge is but Half-yard wide?

Here remember, that your first and third Numbers must be of one Name; therefore call the Half-yard 2 qrs. and state the Question as follows:

$$\begin{array}{r} \text{qrs.} \quad \text{yards.} \quad \text{qrs.} \\ \text{If } 3 \quad \text{—} \quad 60 \quad \text{—} \quad 2 \\ \quad \quad \quad 3 \\ \quad \quad \quad \hline \end{array}$$

$$\begin{array}{r} 2) \quad 180 \\ \hline \end{array}$$

Ans^w. 90 Yards.

*Again: If in 12 Months 100l.
Principal*

Principal gain five Pounds Interest,
what Principal will gain the same In-
terest in 5 Months?

	M.	l.	M.
If	12	— 100	— 5
		12	

$$\begin{array}{r} 5 \overline{) 1200} \\ \underline{} \end{array}$$

Ans. 240 l.

Once more: If 6 Men dig a Trench
24 Days, in what Time would 18
Men do the same Work?

Men.	Days.	Men.
If	6 — 24	— 18
	6	

$$\begin{array}{r} 18 \overline{) 144} \\ \underline{} \end{array}$$

Ans. 8 Days.

2. How

2. How are we to know whether a Question belongs to the *Direct* or the *Indirect* Rule of Three?

A. If the third Number is more than the first, and requires more than the second to answer the Question; or if it is less than the first, and requires less than the second, then it is *Direct*. But if the third Number is more than the first, and requires less than the second, or being less than the first requires more than the second, then it is *Indirect*.—After all, there is no Necessity for puzzling the Learner with this Distinction; for as the Method of stating a Question is the same whether the Proportion be *Direct* or *Indirect*, so the following Rule for the Operation will be sufficient in all Cases, viz. Consider whether the Answer to the Question will be more or less than the second Number; if *more*, the *less* of the two Extremes (that is, of the first

and third Numbers) must be the Divi-
 er; if *less*, the *greater*.

Q. Which way do you prove your
 Work in the *Rule of Three*?

A. It may be proved by a contrary
 ating and Operation, as has been al-
 dy intimated; but the easiest Me-
 od, in Questions that are *Direct*, is
 multiply the first and fourth Num-
 ers together, and the second and
 third, and if the two Products are a-
 ke the Work is right. In Questions
 at are *Indirect*, if the Product of the
 first and fourth multiplied together
 agree with that of the first and second,
 proves the Truth of the Operation.

Q. What is meant by the *Double*
Rule of Three?

A. It is so call'd because Questions
 it are generally answer'd by *two*
 atings, there being five Numbers
 ven to find out a sixth. For Ex-
 ample:

If

If 100l. Principal gain 5l. Interest in 12 Months, what will 246l. gain in 7 Months?

Here the Business is to find out what Interest 246l. will gain in 12 Months at the given Rate, and from thence what they will gain in 7 Months. The first Stating therefore, and Operation, according to the Rules already prescribed, will be as follows:

Pr. Int. Pr.
If 100 — 5 — 246

5
———
1,00) 12,30
20
———

1,00) 6,00 *Ans. 12l. 6s.*

Observe here, that the Product 246 multiplied by 5 is 1230, which being divided by 100 the Quotient is 12, *i. e.* so many Pounds, and the Remainder

remainder is 30. Now the Value of this Remainder is found by the Rule I have already given for that purpose, namely, multiplying it by 10, and dividing the Product by 100, the first Divisor. This done, the Quotient is 6, which are Shillings; and so 12*l.* 6*s.* is the Answer to this Stating of the Question.

As to the second Stating, take notice, that the Answer to the first Stating is always the Middle Number of the last; and therefore 12*l.* 6*s.* being placed in the Middle, and the given Months on the Right and Left according to the Rule, they will stand thus:

Mo.

[142]

Mo. l. s. Mo.
If 12 — 12 : 6 — 7

20

246

7

12) 1722

2,0) 14,3 - 6d.

Ans^w. 7l. 3s. 6d.

Here the middle Number is multiplied by 20 to bring it into Shilling and these being multiplied by 7 the lesser Extreme, and divided by 12 the greater, are reduced to Pounds by dividing by 20, the Quotient being 7l. 3s. 6d. which is the Interest that 246l. will gain in 7 Months at the Rate of 5 per Cent. per Annum.

N.

N. B. The foregoing Question, and others of the same Nature, may be answer'd at one Stating of the 5 Numbers according to the following Directions. Let the three Numbers that contain the Supposition stand first, and after them the two that contain the Demand: And this must be done in such manner that the first be of the same Denomination with the fourth, the second of the same with the fifth, and the third with the Answer required. When the Numbers are thus stated, multiply the two first together for a Divisor, and the three last for a Dividend, and the Quotient will be the Answer in the same Name with the middle Number. I shall put down the State of the former Question, and the Beginning of the Work, the finishing of which I leave for the Exercise of the Learner.

If

	<i>Pr.</i>	<i>M.</i>	<i>Int.</i>	<i>Pr.</i>	<i>M.</i>
If	100	— 12	— 5	— 246	— 7
	12			5	
	—			—	
	1200			1230	
	—			7	
				—	
				1200)	8610

Here the two first Numbers multiplied together produce 1200 for a Divisor, and the three last 8610 for a Dividend, which being rightly divided, and the Remainders that come out in the Operation being again multiplied and divided according to the Rule, the Answer to the Question will be found to be the same as above, *v.* 7*l.* 3*s.* 6*d.*

In Questions where the Proposition is *Indirect*, place your Numbers so that your second and fourth may be of one Denomination, and your third

and fifth. Then multiply your first, second, and fifth Numbers together for a Dividend, and your third and fourth for a Divisor.

For Example: *If 2 Men reap 12 Acres in 6 Days, how many Men will reap 192 Acres in 24 Days?*—The relating and working of this Question is as follows:

<i>Men.</i>	<i>Days.</i>	<i>Acr.</i>	<i>Days.</i>	<i>Acr.</i>
If 2	—6—	—12—	—24—	—192
2		12	12	
—		—	—	—
12		288	)2304(8	
			2304	
			—	
			0	
			—	

Here, according to the Rule, the first, second, and fifth Numbers multiplied into each other give 2304 for Dividend; which being divided by

L

288

288, the Product of the third and fourth Numbers, quotes 8, the Answer to the Question.

Having sufficiently explain'd the *Rule of Three*, I now proceed to the *Rules of Practice*.

CHAP. VIII.

Of PRACTICE; and of TARE and TREET.

2. **W**HAT do you mean by *Rules of Practice*?

A. In effect they are only certain compendious Methods of working the *Rule of Three*; but being of excellent Service for the quick Dispatch of Business, and generally made use of by Merchants and Tradesmen in calling up the Price of their Commodities, they are not improperly call'd *Rules of Practice*.

2.

Q. In what Cases are they chiefly used?

A. In such Questions of the *Rule of Three* as have an *Unit* for their first Number; that is, when the Price of one Thing is given to find the Price of many: And to work Questions by these Rules, it is necessary to have in Memory the *Aliquot Parts* of a Pound and Shilling.

Q. What is meant by *Aliquot Parts*?

A. No more than *Even Parts*, or such Parts of any Number or Quantity as being-taken certain Times are exactly equal to the Whole. Thus 3 is an *Aliquot* or *Even Part* of 12, because being taken four times it is just equal to 12; *i. e.* it is one *Fourth* of that Number.

Q. Which are the even Parts of a Shilling?

A. Learn them from the following Table.

d.					<i>Of a Shilling.</i>
1	is	$\frac{1}{12}$	} that is,	}	one Twelfth
$1\frac{1}{2}$	—	$\frac{1}{8}$			one Eighth
2	—	$\frac{1}{6}$			one Sixth
3	—	$\frac{1}{4}$			one Fourth
4	—	$\frac{1}{3}$			one Third
6	—	$\frac{1}{2}$			one Half

Q. What is the Use of this Table?

A. When any of these even Parts of a Shilling is the Price of one Yard, Ell, Pound, &c. divide the given Number (of which you would know the Price) by such Part, and the Quotient will be the Answer to the Question in Shillings.—An Example or two will make this plain.

Suppose you would know the Price of 672 Oranges, at the rate of 1 Penny an Orange. You see by the Table that a Penny is one 12th Part of a Shilling, therefore divide 672 by 12 and the Quotient is the Answer:

$$12) 672$$

Answer 56 Shillings.

Again: *What would the same Number cost at three Half-pence an Orange?*
 -- The Table shews you that 1 d. $\frac{1}{2}$ is one 8th Part of a Shilling, therefore divide 672 by 8, and you will have the Answer:

$$8) 672$$

Answer 84 Shillings.

or 4l. 4s.

N. B. By cutting off the last Figure of any Number of Shillings with a Dash, and halving the Figures on the Left, (which is the same thing as dividing by 20) you have the Number of Pounds; and the Figure on the Right Hand of the Dash is Shillings, as you see in the Example above. And if when you have halv'd the

L 3

Left-

Left-hand Figures there be a Remainder, it is 10 Shillings, and as such must be reckon'd with the separated Figure, as in the next Example.

*At 2 Pence the Yard, what will 672 Yards cost?—*Two-pence, as you see in the Table, is one 6th Part of a Shilling, therefore divide by 6, and the Quotient gives the Number of Shillings.

$$\begin{array}{r} 6 \overline{) 672} \\ \hline \end{array}$$

Answer $11\frac{1}{2}$ Shillings.

or $5\text{ l. } 12\text{ s.}$

Here, I say, the Half of 11 is 5, and 1 over, which being reckon'd with the Figure on the Right-hand of the Dash makes 12, and so the Answer to the Question is 5 Pounds 12 Shillings.

After this Method may any Question be resolved where the Price is an even

even Part of a Shilling; but if the Price be not an even Part, as 5*d.* or any Number of Pence between 6 and 12, it will require more than one Division, as will be seen in the following Examples.

At 5 d. an Ell, what will 672 Ells cost?—Here consider that Five-pence contains two even Parts of a Shilling, *viz.* 3 Pence and 2 Pence; 3 Pence being $\frac{1}{4}$, and 2 Pence $\frac{1}{6}$ Part: Therefore divide 672 by 4 and by 6, and add the two Quotients together, the Total will be the Answer.

$$\begin{array}{r}
 672 \\
 d. \quad \hline
 3 \frac{1}{4} \quad 168 \\
 2 \frac{1}{6} \quad 112 \\
 \hline
 \end{array}$$

Answer 28|0 Shillings.

or 14*l.*

L 4

N. B.

N. B. It would have been the same thing to have divided by 3 and 12, because 4 Pence and 1 Penny (which are equal to 5 Pence) are $\frac{1}{3}$ and $\frac{1}{12}$ Parts of a Shilling.—Or if you first divide by 3 for the Groats, and then the Quotient by 4, because 1 Penny is $\frac{1}{4}$ of a Groat, the Answer will be the same, as appears by the Work :

$$\begin{array}{r}
 672 \\
 d. \quad \text{---} \\
 4\frac{1}{3} \quad 224 \\
 1\frac{1}{4} \quad 56 \\
 \text{---} \\
 280 \\
 \text{---}
 \end{array}$$

Again: *At 9 Pence a Peck, what will 144 Pecks cost?*—Here 6 and 3, which are equal to 9 Pence, are the even Parts of a Shilling; therefore for 6 Pence take $\frac{1}{2}$ the given Number, and for 3 Pence $\frac{1}{4}$, and the Work will stand as follows:

[153]

$$\begin{array}{r}
 144 \\
 d. \text{ ---} \\
 6\frac{1}{2} \quad 72 \\
 3\frac{1}{4} \quad 36 \\
 \text{---}
 \end{array}$$

Answer 10.8 Shillings.

or 5 l. 8 s.

If the Price be 10 Pence, you may take half the given Number for 6*d.* and divide by 3 for the 4*d.* because 6*d.* is Half, and 4*d.* one Third of a Shilling.—Or you may annex a Cypher to the Right of the given Number, (which is the same as multiplying by 10) and it thereby becomes Pence; which are brought into Shillings and Pounds by dividing by 12 and 20. For Example:

*What will 126 lb. of Hops cost at 10*d.* per Pound?—Annex a Cypher to*
126,

126, and the Work will appear as follows:

$$12 \overline{) 1260}$$

$$\underline{\hspace{1cm}}$$

$$105$$

$$\underline{\hspace{1cm}}$$

Answer

5 l. 5 s.

*At 11 d. 1 q. a Yard, what will 752 Yards cost?—*Here you begin by taking one Half of the given Number for Six-pence, then Half of that for Three-pence, Half of that again for three Half-pence, and Half of that for three Farthings, which make just 11 d. 1 q. And these Sums added together, and divided as before, give the Answer to the Question, as you will see by the Work at length.

[155]

$$\begin{array}{r}
 6d. \frac{1}{2} \quad \underline{752} \\
 376 \\
 188 \\
 94 \\
 47 \\
 \hline
 705
 \end{array}$$

Answer 35l. 5s.

Take one Example more, wherein the Price is Pence and Farthings. —

At 3d. $\frac{3}{4}$ an Ounce, what will 324 Ounces cost?

$$\begin{array}{r}
 324 \\
 3d. \frac{3}{4} \quad \underline{} \\
 \frac{3}{4} \quad \frac{3}{4} \quad \underline{} \quad 81 \\
 20 \quad \text{—} \quad 3 \\
 \hline
 101
 \end{array}$$

Answer 5l. 1s. 3d.

Here,

Here, because 3 *d.* is $\frac{1}{4}$ of a Shilling, I divide the given Number by 4, and the Quotient is 81; which I also divide by 4, because three Farthings is $\frac{1}{4}$ of Three-pence, and the Quotient is 20, and 1 over. The Remainder I consider is one Three-pence, and therefore I set down 3 and adding the two Quotients together the Total is 101, which being divided by 20 gives the Answer as above.

I shall now give an Example or two wherein the Price is a Shilling, and some Number of Pence and Farthings. In which Case the Rule is; to let the top Line (*i. e.* the given Number) stand for the Shilling, and take the Parts as before, for the remaining Pence.

At 13 d. $\frac{1}{2}$ a Yard, what will 144 Yards cost?—Here it is plain they will cost 144 Shillings, and as many Pence and Half-pence; wherefore the given Number is reckon'd as Shillings,

and

and $\frac{1}{8}$ of it taken for the 1 d. $\frac{1}{2}$, which added together give 162 Shillings, equal to 8 l. 2 s. the Answer.

$$\begin{array}{r}
 144 \\
 18 \\
 \hline
 162 \\
 \hline
 \text{Answer} \quad 8 \text{ l. } 2 \text{ s.} \\
 \hline
 \end{array}$$

At 16 d. $\frac{1}{2}$ a Yard, what will 143 Yards cost?—See the Work at length.

$$\begin{array}{r}
 143 \\
 35-9 \\
 17-10\frac{1}{2} \\
 \hline
 19|6-7\frac{1}{2} \\
 \hline
 \text{Answer} \quad 9 \text{ l. } 16 \text{ s. } 7 \text{ d. } \frac{1}{2} \\
 \hline
 \end{array}$$

Here the given Number stands for the Shillings as before, the Reason of which

which is evident; then I divide it by 4 for Three-pence, and the Quotient is 35 and 3 over, which being so many Three-pences I set down 9: Then for the remaining $1d. \frac{1}{2}$ of the given Price, I take half that Quotient, which is 17 and 1 over; and this 1 being reckon'd as a Shilling, and added to the 9 Pence, makes 21 Pence, the Half of which is $10d. \frac{1}{2}$. These Sums being added together, and divided give the Answer as express'd in the Work.

I now proceed to Questions where in the *Aliquot* or *Even* Parts of Pound are to be consider'd, which you may learn from the following Table.

<i>s.</i>	<i>d.</i>		<i>Of a Pound.</i>
1	0	is $\frac{1}{20}$	one 20th
1	8	— $\frac{1}{12}$	one 12th
2	0	— $\frac{1}{10}$	one 10th
2	6	— $\frac{1}{8}$	one 8th
3	4	— $\frac{1}{6}$	one 6th
4	0	— $\frac{1}{5}$	one 5th
5	0	— $\frac{1}{4}$	one 4th
6	8	—	one 3d
10	0	— $\frac{1}{2}$	one half

} that is, {

In Questions where the Price is 1*s.* only, you have nothing to do but to divide by 20, which you know is done by cutting off the Right-hand Figure of the given Number for Shillings, and taking Half of the Left-hand Figures for Pounds. In like manner, if the Price be 1*s.* 8*d.* divide by 12; if it be 2*s.* divide by 10; if it be 2*s.* 6*d.* divide by 8; and so of any other even Part of a Pound; which being learnt from the Table, and

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and the Operation so easy, Example of this Kind would be superfluous.

If the Price does not consist of even Parts, divide it into such, and the Sum of them will be the Answer to the Question. For Example: *At 10 Shillings an Ell, what will 288 Ells cost?*

	288	
$\frac{1}{10}$ for 2s.	28l.	16s.
$\frac{1}{20}$ for 1s.	14l.	8s.
<i>Ans-w.</i>	43l.	4s.

Here observe, that after dividing by 10 the Remainder is 8, which are 8 *Tenths* of a Pound, that is, 16 Shillings; and after dividing by 20 the Remainder is also 8, but then it is *Twentieths* of a Pound, that is, 8 Shillings.—The Learner must always con-

fide

Consider well the Value of a Remainder,
or he will fall into frequent Mistakes.

The same Question (and others of
the like Nature) may be answer'd as
easily by multiplying the given Num-
ber by the Number of Shillings in the
Price, and then dividing by 20, as
you see in this Instance:

$$288$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$86\frac{1}{4}$$

Ans^w. 43 l. 4 s.

*At 2 s. 4 d. $\frac{1}{2}$ a Pound, what will
41 Pounds cost?*

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$$\begin{array}{rcl}
 \frac{1}{10} \text{ for } 2s. & 14 - 2 - & 0 \\
 \frac{1}{6} \text{ of } 2s. \text{ for } 4d. & 2 - 7 - & 0 \\
 \frac{1}{8} \text{ of } 4d. \text{ for } 2q. & 0 - 5 - & 10\frac{1}{2} \\
 \hline
 \text{Answ.} & 16 - 14 - & 10\frac{1}{2} \\
 \hline
 \end{array}$$

In this Example, after dividing by 10, there remains 1, which being Tenth of a Pound, that is, 2 Shillings I accordingly set down 2, and the Line stands as 14 Pounds 2 Shillings. Then dividing 14 by 6 I find a Remainder of 2, which being Pounds consider as 40 Shillings, and joining them to the first-remaining 2, I say the 6's in 42 are 7, and therefore put down 7 in the Place of Shillings. The Line then is 2*l.* 7*s.* or 47 Shillings one Eighth of which is 5*s.* 10*d.* $\frac{1}{2}$. And these three Sums being added together

gether give the Answer to the Question, as above express'd.

Once more: *What will 375 Bundles of Paper cost, at 17s. 6d. each Bundle?*

	375	
	—	
$\frac{1}{2}$ for 10s.	187	— 10 — 0
$\frac{1}{2}$ of 10s. for 5s.	93	— 15 — 0
$\frac{1}{2}$ of 5s. for 2s. 6d.	46	— 17 — 6
	—	—
<i>Ans^w.</i>	328	— 02 — 6
	—	—

The Manner of working in this Example will be easily understood by a due Attention to the last, and therefore needs no particular Explanation. But it may not be amiss to observe, that the above Question, and others of the same Sort, consisting of Shillings and Pence, may be answer'd another Way, which in some Cases

may be more convenient and easy,
viz. by multiplying the given Num-
 ber by the Number of Shillings, and
 then working for the Pence as before.
 For Example:

*At 9s. 9d. a Yard, what will 141
 Yards cost?*

	141
Multiply by	9
	1269
$\frac{1}{2}$ of 141 for 6d.	70 - 6
$\frac{1}{2}$ of that for 3d.	35 - 3
Total	1374 - 9
Answ.	68 - 14 - 9

In this Case it would have been the
 same thing, after multiplying by
 for the Shillings, to have divided the
 Product by 12 for the Nine-pence
 because

because 9 Pence are a 12th Part of 9 Shillings. And so there is scarce any Question but may be answer'd by different Operations, as the ingenious Scholar will find by his own Observation.

When the Price is any thing between one Pound and two, let the given Number stand for the Pound, and take Parts for the Shillings and Pence. For Example:

At 1l. 2s. 6d. per Quarter, what will 286 Quarters of Malt cost?

$$\begin{array}{r} 286 \\ \frac{1}{8} \text{ for } 2s. 6d. \quad 35-15 \\ \hline \end{array}$$

Ans^w. 321l. 15s.

If the Price consists of Pounds, Shillings and Pence, multiply the given Number by the Number of Pounds, and for the Shillings and Pence work

M 3

by

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by the former Rules. For Example:

At 2l. 17s. 6d. the Hundred, what will 144 Hundred cost?

For 2l. mult. by	144	
	2	
	288	
$\frac{1}{2}$ of 144 for 10s.	72	
$\frac{1}{2}$ of that for 5s.	36	
$\frac{1}{2}$ of that for 2s. 6d.	18	
Answ.	414l.	

When with the given Quantity (suppose a Hundred wt.) there are any odd Parts, as $\frac{1}{4}$, $\frac{1}{2}$, &c. such Parts must be taken from the given Price in the Course of the Operation; as may be understood by the following Example.

At 5l. 5s. the Hundred, what will 365 C. $\frac{1}{2}$ cost?—Here, for the Pounds,
I mul.

I multiply the given Number by 5, then take $\frac{1}{4}$ of it for the 5 Shillings, and $\frac{1}{2}$ of the given Price for the $\frac{1}{2}$ Hundred, and the Sum Total gives the Answer to the Question, as appears by the Work at length.

For 5 <i>l.</i> mult. by	365	
	5	
	1825	
$\frac{1}{4}$ of 365 for 5 <i>s.</i>	91	— 5
$\frac{1}{2}$ of given Price	2	— 12 — 6
<i>Ans.</i>	1918 <i>l.</i>	17 <i>s.</i> 6 <i>d.</i>

N. B. If the odd Parts had been $\frac{3}{4}$, after taking half of the given Price I must have taken half of that, *viz.* 1*l.* 6*s.* 3*d.* — And hence you may judge how to proceed in Cases of the like Nature.

By the foregoing Rules of *Practice*

all Sorts of foreign Coin may be reduced to Pounds Sterling, of which I need only give one Example.

What is the Amount of 716 Moidores at 1l. 7s. each? — Here the given Numbers stand for the Pound, $\frac{1}{4}$ of which must be taken for 5s. and $\frac{1}{10}$ for 2s. and all added together are the Answer to the Question.

	716
$\frac{1}{4}$ for 5s.	179
$\frac{1}{10}$ for 2s.	71 — 12
<i>Ans^w.</i>	966l. — 12s.

As it is impossible for me to speak of the various and almost innumerable Methods and Rules of Practice, I have chosen such as I thought were most intelligible and useful; to which I shall only add two or three very compendious Ways of working in some Cases,

Cases, which deserve the Learner's Notice.

In any Question when the given Price is an even Number of Shillings, multiply the given Quantity by half that Number, doubling the Units of the first Product for Shillings, and the other Figures of the Product will be Pounds. For Example :

What will 276 Ells of Holland cost, at 8s. an Ell?—Proceeding according to the Rule, the Work will appear as follows :

$$\begin{array}{r} \text{Multiply } 276 \\ \text{by } \frac{1}{2} \text{ of } 8 \quad 4 \\ \hline \end{array}$$

Answer 110l. 8s.

Here I multiply the given Number of Ells by 4, which is half the Number of Shillings in the given Price, saying, 4 times 6 is 24; but instead of putting down 4, the Units of the Product,

Product, I put down twice as many *viz.* 8, which are the Shillings in the Price required. Then I proceed as usual, saying, 4 times 7 is 28, and that I carry is 30, set down 0 and carry 3; 4 times 2 is 8, and 3 is 11, which being put down gives the Number of Pounds requir'd, *viz.* 110, and the Operation is finish'd.

If the Price be an odd Number of Shillings, work for the even Number as before, and for the odd Shilling take $\frac{1}{20}$ of the given Number, and add to the Product. For Example:

At 17s. a Yard, what will 17 Yards cost?—Observing your Directions, your Work and Answer will be as follows;

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Mult. for 16s. by $\begin{array}{r} 172 \\ 8 \end{array}$

Product $\begin{array}{r} 137-12 \\ \hline \end{array}$

$\frac{1}{20}$ for 1s. $\begin{array}{r} 8-12 \\ \hline \end{array}$

Answer $\begin{array}{r} 146l. 4s. \\ \hline \end{array}$

If the Price be 2 Shillings, the Question is answer'd, by doubling the Units (*i. e.* the Right-hand Figure) of the given Number, which are to be reckon'd as Shillings, and the rest of the Figures will be Pounds. For Example:

596 Geese at 2s. each

will cost 59l. 12s.

Q. How are Sums in *Practice* to be proved?

A. As they may be wrought different Ways, the one may be a Proof of

of the other; or they may be proved by the *Rule of Three*.

Of TARE and TRETT.

After laying down the Rules of *Practice*, it is usual for Authors to speak of TARE and TRETT, which are Allowances made by Merchants in selling their Commodities.

Tare is an Allowance made to the Buyer for the Weight of the Case, Bag, Chest, &c. in which the Goods are contain'd; and this is either regulated by Custom, or by a particular Agreement between the Buyer and Seller.

Trett is an Allowance of 4 lb. for 104 lb. made in several Sorts of Goods for Waste, Dust, &c. after the *Tare* has been deducted.

There is also another Allowance of 2 lb. for every 3 Cwt. call'd CLOUGH or CLOUGH, which is sometimes made

to Retailers for the Turn of the Scale, after the Deduction of the former Allowances.

N. B. The whole Weight, before any of these Allowances are made, is call'd *Gross*; but when they are deducted, or so many of them as are customary, the Remainder is call'd *Neat* or *Nett*.

If the Tare of any Quantity of Goods be so much in the Whole, and no other Allowance, the *Neat Weight* is found by subtracting the Tare from the *Gross Weight*; but if the Tare be at so much *per Chest*, &c. multiply the Pounds Tare by the Number of Chests, subtracting as before; and if at so much *per Cwt.* take such Part or Parts of the *Gross Weight*, as the Tare is of a Hundred.

In order to work Questions of this Kind by the Rules of Practice, it is necessary to remember the *even Parts* of

of a Hundred *wt.* (that is 112 *lb.*) and
of a Quarter, which are as follow:

lb.

7	is	$\frac{1}{16}$	} of a C.
8	—	$\frac{1}{14}$	
14	—	$\frac{1}{8}$	
16	—	$\frac{1}{7}$	

1	is	$\frac{1}{28}$	} of a Qr.
2	—	$\frac{1}{14}$	
4	—	$\frac{1}{7}$	
7	—	$\frac{1}{4}$	
14	—	$\frac{1}{2}$	

As the *Trett* is always 4 *lb.* per 100
the Method of finding it is to divide
the Weight by 26 after the Tare
deducted, because 4 times 26 make
104; and then subtracting it, the Re-
mainder is the *Neat Weight* required.

Clough (which is 2 *lb.* for 3 C.) may
be found by dividing the Line from
whence it is to be deducted by 168
because 2 *lb.* is the 168th Part of

36 lb. or 3 Hundred Weight.

Having laid down these Rules, I shall only give three plain Examples of the Manner of subtracting *Tare*, and finding out the *Nett Weight* of any Quantity of Goods; leaving the Deduction of *Tret*, *Clough*, or whatever Allowances may be customary, to the scholar's own Ingenuity.

If 15 C. 2 qrs. 13 lb. *Tare* be allow'd in 456 C. 1 qr. 19 lb. of Tobacco, what will be the *Neat Weight*?

	C.	qrs.	lb.	
from	456	— 1 —	19	Gross
subtract	15	— 2 —	13	Tare.
Rem.	440	— 3 —	6	Neat.

Here I say, 13 from 19 and there remains 6; 2 from 1 I cannot, but 2 from 5 (borrowing 4 qrs. that is, 1 C.) and there remains 3; then, paying the C. that I borrow'd, I say, 1 and

5 is 6, which take from 6 and 0 remains, 1 from 5 and there remains 4; 0 from 4 and there remains 4; and the Work is finish'd, the Remainder being the Answer to the Question.

Again: *What is the Neat Weight of 3 Frails of Raisins, weighing all together 10 C. 3 qrs. 2 lb. Gross, the Tare 20 lb. per Frail?*

C.	qrs.	lb.		lb. per Frail.
10	3	2	Gross.	20
	2	4	Tare.	3
10	0	26	Neat.	60 or 2 qrs. 4 lb.

Having first found the Tare by multiplying the Pounds allow'd on each Frail by the Number of Frails, viz. 3, I set it down under the Gross Weight, and subtract as before, saying, 4 from 2 I cannot, but 4 from 30 (borrowing 28 lb. or 1 qr.) and there remains 26; then, 1 that I borrow

and 2 is 3, which take from 3 and 0 remains; lastly, 0 from 10 and there remains 10; which being set down shews the Neat Weight 10 C. 26 lb. as above express'd.

Once more: Of 246 C. 3 qrs. 12 lb. Gross, Tare 14 lb. per Hundred, what will be the Neat Weight?

C. qrs. lb.

246—3—12 Gross.

$\frac{1}{8}$ for 14 lb. 30—3—12 Tare.

216—0—0 Neat.

Here, according to *Practice*, the Pounds Tare per C. being an even Part of a C. namely, one Eighth, I divide the Gross Weight by 8, which gives the Tare of the whole Quantity. In doing this I say, the 8's in 24 are 3, therefore set down 3; the 8's in 6 none, and therefore setting down a Cypher, I consider how many Quarters the 6 C. contain, which

N

are

are 24, and these carried to the 3 make 27. Then I say, the 8's in 27 are 3, which I set down, and the Number of Pounds contain'd in the overplus 3 *qrs.* I add to the 12 Pounds, making 96 in all; and there being just 12 times 8 in 96, I set down 12, and find the Tare to be 30 C. 3 *qrs.* 12 *lb.* as above express'd. This being subtracted from the *Gross*, the Remainder is 216 C. the *Neat Weight* required.

I hope these few Examples will be sufficient to give the Scholar a Notion of the Method of *subtracting* and *dividing* in Questions of this Nature, which are the two chief Operations in finding the *Tare*, *Trett*, or other Allowance in any Quantity of Goods, and consequently the *Neat Weight* after such Deductions.

Here, in compliance with Custom I might add several other Rules, as

Barter

Barter, Fellowship, Profit and Loss, Alligation, &c. But as Questions in all these Rules are answer'd either by the *Rule of Three*, or the *Rules of Practice*, I think they would be an useless if not a burdensome Addition to our little Treatise; for he who thoroughly understands the *Rule of Three*, and those of *Practice*, will scarce find any Difficulty in solving whatever Questions are necessary in Trade or the common Affairs of Life. However, I shall say something of another Rule, which, though of no great Use, may be an agreeable Amusement to the young Scholar, and serve to exercise his Ingenuity; I mean the *Rule of Position*.

CHAP. IX.

Of POSITION, or the Rule of FALSE.

2. WHY is this Rule call'd *False*?

A. Not from its being in itself really erroneous, but because we make use of *false* supposed Numbers to find out the true Numbers sought.

2. Why is it call'd *Position*?

A. It is the same as *Supposition*, from the *supposed* Numbers.

2. Is it not divided into two Parts?

A. Yes; it is usually divided into two Parts, *Single* and *Double*.

2. What is the Nature of *Single Position*?

A. In the *Single* Rule we use but one supposed Number to find out the Truth, as in the following Example:

Three Persons, A, B, C, bought a Quantity of Lead, to the Value of 140.

of which **B** paid as much again as **A**, and **C** as much again as **B**. How much did each pay?

Suppose **A** paid 10*l*. then **B** must pay 20*l*. and **C** 40*l*. the Total of which is 70*l*. but should be 140*l*. —Now as the Total arising from the Error is to the true Total, so is the supposed Part to the true Part. Therefore, in this Example, as 70*l*. is to 140*l*. so is 10*l*. to the Sum that **A** really paid, which of Consequence must be 20*l*. for 70 is the Half of 140, and 10 is the Half of 20. But the Proportion of the several Numbers to each other being not always discernible at Sight, we must work in Questions of this Kind by the *Rule of Three*, in order to obtain the Answer. In the present Case therefore I say, If 70*l*. should be 140*l*. what should 10*l*. be? Which I work as follows:

N 3

If

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If 70—140—10

10

70)1400

20

Here, multiplying the second and third Numbers together, and dividing the Product by the first, according to the *Rule of Three*, the Quotient is 20, which is the Number of Pounds that *A* is paid, as appears by setting down the several Shares as the Question requires.

20 for *A*.

40 for *B*.

80 for *C*.

Total 140

Take

Take one Example more: A Man overtaking a Maid, who was driving a Flock of Geese, said to her, Good-morrow, Sweetheart, whither are you going with your 99 Geese? Sir, said she, you mistake the Number; for if I had as many more, and half as many more, and one fourth Part as many, then I should have but 99. The Question is, how many Geese she had?

Let us suppose she had 40, then as many, and half as many, and one Fourth as many, will make 110, which should have been 99. Therefore say, If 110 should be 99, what should 40 be? Or, If 110 come from 40, what will 99 come from? For it matters not whether 40 or 99 is the middle Number in Stating, since they are to be multiplied together. See the Work at length:

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If 110—40—99
40

110)3960(36
330
660
660
0

Here the Quotient is 36, which will appear to be the Number of the Woman's Geese, by setting down the several Numbers according to her Calculation.

	36 Geese.
As many more	36
$\frac{1}{2}$ as many	18
$\frac{1}{4}$ as many	9
	—
Total	99
	—

2. What

Q. What is the Nature of *Double* Position?

A. When there is no Partition in the Numbers to make a Proportion, the *Double Rule* is used, wherein we make *two* Suppositions; and if with either of them we find the Numbers that solve the Question, there is no more to be done; but if, as it commonly happens, we err in both, set down the Suppositions, and over-against them their respective Errors, mark'd thus + if too much, and thus — if too little. Then multiplying them cross-ways, that is, the first Supposition by the second Error, and the second Supposition by the first Error; if both the Errors are alike, *i. e.* both too little, or both too much, subtract the lesser Product from the greater, and divide the Remainder by the Difference of the Errors; but if the Errors are unlike; *i. e.* one too little, and the

the other too much, the Sum of these Products must be divided by the Sum of the Errors: In either Case the Quotient will give a true Answer to the Question, as will appear by the following Examples.

Three Merchants, A, B, C, built a Ship, which cost 1600l. of which A paid a Sum not known, B paid as much again as A within 50l. and C as much as A and B within 100l.—The Question is, What did each Man pay?

Now suppose *A* paid 200l.

Then *B* paid ——— 350

And *C* paid ——— 450

—————
Total 1000

But it should have been 1600, therefore the Error is 600, too few.

Sup-

Supposing again that *A* paid 250*l*.

Then *B* paid 450

And *C* paid 600

Total 1300

Which Sum is still too little by 300. Therefore setting down the Suppositions and their Errors in the Manner before directed, they will stand thus:

<i>Suppositions</i>	<i>Errors.</i>
200 <u> </u>	600
250 <u> </u>	300

Then multiplying the first Supposition by the second Error, the Product is 60000; and the second Supposition multiplied by the first Error produces 150000; the lesser of which Products being subtracted from the greater, the Remainder is 90000; and this being divided by 300, the Difference of the Errors, quotes 300, the Sum that *A* paid,

A. paid, as will appear by adding to it the Shares of *B* and *C* according to the Question.

A paid ——— 300

B paid ——— 550

C paid ——— 750

Total 1600

In this Example the Errors were both alike, *i. e.* both too little; in the following they are unlike, *i. e.* one too little, the other too much.

*When first the Marriage-Knot was
ty'd*

*Betwixt my Wife and me;
My Age did hers as far exceed
As three times three does three:
But after ten and half ten Years
We Man and Wife had been,
Her Age came up as near to mine
As eight is to sixteen.*

— Nov

— Now pray,
What were our Ages on the Wed-
ding-Day?

First, suppose the Wife's Age to be 21 Years, the Husband's must be 63; then adding 15 to each, her Age becomes 36, and his 78. This Error therefore is 3 Years too few.

Again: Suppose she was 9 Years old, he must have been 27; and by adding 15 to each, her Age is made 24, and his 42. Here the Error is 3 too many.

Therefore setting down the Suppositions and their Errors, with their respective Marks, as above directed, they appear thus:

Suppositions. Errors.

21 — 3

9 + 3

Then multiplying cross-ways, 3 times 21 is 63, and 3 times 9 is 27,
these

these Products added together (because the Errors are unlike) make 90 for a Dividend; which divided by 6, the Sum of the Errors, quotes 15, the Wife's true Age when married, as is thus proved:

She being	15,	he must be	45
	15	added to each	15
	—		—

Make her	30,	half of his	60
	—		—

I forbear giving more Examples of this Nature, as the Scholar may easily frame them himself; but I shall add a Question or two in *Progression*, which will not be displeasing to those who take Delight in the Art of Numbers.

CHAP. X.

PROGRESSION *briefly explain'd, with some Examples.*

Q. IS not this Rule divided in two Parts?

A. Yes; Progression is either *Arithmetical* or *Geometrical*.

Q. What is *Arithmetical Progression*?

A. It is the regular Increase or Decrease of any Series of Numbers by the continual Addition or Subtraction of some equal Number. So 1, 3, 5, 7, 9, and 35, 28, 21, 14, 7, are two Ranks of Numbers in *Arithmetical Progression*; the first increasing by the continual Addition of Two, and the second decreasing by the continual Subtraction of Seven.

Q. What is *Geometrical Progression*?

A. It

A. It is the Increase or Decrease of any Series of Numbers by an equal *Ratio*, that is, by the continual Multiplication or Division of some equal Number. Thus 2, 4, 8, 16, increase in Geometrical Progression by a *double Ratio*, or continual Multiplication by 2; and 135, 45, 15, 5, decrease in the same Manner in a *triple Ratio*, or a continual Division by 3.

2. What is chiefly to be regarded in *Progression*?

A. The five following Things, *viz.*
 1. The first Term; 2. The last Term;
 3. The Number of Terms; 4. The *Ratio*, or equal Difference; 5. The Sum of all the Terms.—By knowing some of these Things the rest are found, according to various Rules laid down by Arithmeticians, too numerous to be here transcribed and exemplified; and therefore we refer the
 Curious

Curious to larger Treatises for Satisfaction in these Particulars, which would swell our little Work beyond its intended Limits. However, we shall give the few Examples promised, and so conclude.

A Grazier having brought 23 fine Oxen to Market, a Butcher offers him 16l. apiece, and take them all. The Grazier refuses the Money, but says he, If you'll give me what the last Ox will come to, reckoning the first at a Farthing, the second at a Halfpenny, the third at a Penny, and so doubling the Price through the whole Number, you shall have them: To which the Butcher readily assents, thinking he had made an excellent Bargain.—The Question is, What did the Butcher pay according to this Agreement for the 23 Oxen?

Here observe, that the first Farthing is to increase geometrically in a double Ratio, that is by 2, till the

O Number

Number of Terms amount to 23. But as it would be a troublesome and tedious Operation to multiply the several Terms by 2 from the first to the last, in order to find out the Price required, set down a few of the leading Terms, and place their Exponents over them in the following Manner;

0	.	1	.	2	.	3	.	4	.	5
1	.	2	.	4	.	8	.	16	.	32

The upper Row of Figures, call'd *Exponents*, serve to shew the Distance of the Terms from Unity, or from the first Term of the Series: But observe, that as the Exponents are less by one than the Terms, the Term we are in search of, *viz.* 23, will answer to the Exponent 22.—Now the Rule for finding any distant Term, after a few of the leading ones are set down, is this: Multiply the last-found Term by itself, and it will produce a Term double thereto; which again multiplied by
itself

itself will produce another double to the last: And thus proceed till either you produce the Term sought, or one a little short of it; which may be compleated by multiplying it again by that Term which stands under such Exponent as will make up the Number.

In the present Case therefore you are to multiply the 32 by itself, which being the 5th Term (according to the Exponent) produces the 10th, namely, 1024; and this again multiplied by itself gives 1048576, the 20th Term from the first, but taking the first into the Number it is the 21st Term: Therefore as two Terms are wanting to make up the Number required by the Question, *viz.* 23, the last Product is to be multiplied by the Term under the Exponent 2, which is 4, and this gives 4194304, the 22d Term if reckon'd by the Exponents, but in

O 2

reality

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reality the 23d, and consequently the
Price of the Oxen in Farthings, which
makes 4369l. 1s. 4d.—See the Work
at length:

$$\begin{array}{r}
 32 \\
 32 \\
 \hline
 64 \\
 96 \\
 \hline
 1024 \\
 1024 \\
 \hline
 4096 \\
 2048 \\
 10240 \\
 \hline
 1048576 \\
 \hline
 4 \\
 \hline
 4194304 \\
 \hline
 \end{array}$$

Hence

Hence you may learn how necessary it is to have a competent Knowledge of Arithmetic, to prevent being imposed upon by cunning and designing Persons; for had the Butcher been but a little acquainted with the Nature of Numbers, he had not been over-reach'd by the Grazier, and paid such an extravagant Price for his Oxen.

I shall propose another Question of the same Kind, and give the Answer to it, leaving the Scholar to try his Skill in the Operation.

Suppose a Nobleman purchases a fine Horse, having 28 Nails in his Shoes, at the Price of the last Nail, valuing the first at a Farthing, the second at two, the third at four, and so doubling the Number of Farthings for each Nail, till the 28 are reckon'd. The Question is, What was the Value of the last Nail, i. e. the Price of the Horse? — Answer,

swer, 134217728 Farthings, which is
139810*l.* 2*s.* 8*d.*

Take one Question more of a different Nature, *viz.* *How many Changes can be rung upon 10 Bells?*

The Method of answering this is, to set down the whole Number of Terms given, (as 10 in this Example) and they will form a Series of Numbers in *Arithmetical Progression*: Then multiply the first by the second, that Product by the third, and that Product by the fourth, and so on, till you have gone through all the Terms, and the last Product is the Answer sought. And by this means you not only find how many Changes can be rung upon 10 Bells, but upon 2, 3, 4, 5, 6, 7, 8, and 9; as appears by the Work.

Bells.

Bells. *Changes.*

1	—	—	—	—
2	—	—	—	2
3	—	—	—	6
4	—	—	—	24
5	—	—	—	120
6	—	—	—	720
7	—	—	—	5040
8	—	—	—	40320
9	—	—	—	362880
10	—	—	—	3628800

Every one knows that 1 Bell admits of no Variation; but the Changes the other Numbers are capable of are found by multiplying after this Manner: Twice 1 is 2, 3 times 2 is 6, 4 times 6 is 24, 5 times 24 is 120, &c.—To make this plain by one Instance, let *abc* be 3 Bells, and you will find they may be changed 6 Ways with respect to their Places, and no more,

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more, namely, *abc, acb, bac, bca, cab, cba.*

In like Manner, may the Change of any other Number of Things be found, as of 12 Bells, 24 Letters, &c. And this is call'd *Permutation.*

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